

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVIII.

JUNE 14, 1918.

No. 8

Major P. A. MACMAHON, D.Sc., Sc.D., LL.D., F.R.S., PRESIDENT,
in the Chair.

Godfrey Harold Hardy, M.A., F.R.S., Trinity College, Cambridge;

George William Jones, M.A., B.Sc., LL.B., F.R.S.E., Lieut.
R.N.V.R., 4 Bassett Road, London, W. 10;

Francis Everard Lamplough, M.A., c/o Chance, Bros. & Co.
Lighthouse Works, Birmingham;

John Logie, M.A., B.Sc., F.R.Met.Soc., Sub-Lieut. R.N.V.R.,
1 Eversley Terrace, Green Lane, Goodmayes, Essex; and

Mear Rabson, M.A., Senior Mathematical Master, Whitgift
School, Croydon, 7 Burlington Avenue, Kew Gardens,
Surrey,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows
of the Society, the names of the proposers from personal knowledge
being appended :—

Charles Frederick Butterworth, Waterloo, Poynton, Cheshire
(proposed by C. L. Brook);

Mark Herbert Rollason, B.Sc. (Lond.), Science Teacher,
Secondary School, etc., Chester-le-Street, 6 Osborne Road,
Chester-le-Street, Co. Durham (proposed by D. Brunt); and

Dr. Gustaf Strömberg, Solar Observatory, Mount Wilson,
California, U.S.A. (proposed by W. S. Adams).

Frederick Hanley Seares, Solar Observatory, Mount Wilson, California, U.S.A. ;

Harlow Shapley, Solar Observatory, Mount Wilson, California, U.S.A. ; and

Joel Stebbins, Professor of Astronomy, University of Illinois, Urbana, U.S.A.,

were balloted for and duly elected Associates of the Society.

Fifty presents were announced as having been received since the last meeting, and the thanks of the Society were returned to the respective donors.

SPECIAL GENERAL MEETING.

A Special General Meeting, of which due notice had been given to the Fellows, was held the same day, at 6.45 o'clock, Major P. A. MacMahon, D.Sc., Sc.D., LL.D., F.R.S., President, in the Chair.

The following addition to the By-laws, proposed by the Council, was put to the meeting and carried :—

SECTION VIII.A.—Of *Additional Meetings.*

The Council may at any time call additional meetings, when it seems to them necessary, for the purpose of reading Communications relating to Astronomy and the subjects connected therewith.

The Proposed Change in the Astronomical Day. Letter from the Secretary to the Admiralty.

(Communicated by the Secretaries.)

[*Introductory Note.*—A circular was recently drawn up by the Council asking for opinions and suggestions on the subject of the proposed change in the commencement of the astronomical day from noon to midnight. As many of the replies to the circular expressed a desire for a further statement of the wishes of seamen regarding the proposal, the Secretary to the Admiralty, at whose request the circular was originally drawn up, was asked whether such a statement could be issued ; he has now sent the following letter, which has been prepared under the direction of the Lords Commissioners of the Admiralty, and which the Council have decided to print in the *Monthly Notices*.]

1. The advantage to seamen of basing the astronomical data given in the *Nautical Almanac* upon a civil day of 24 hours,

beginning at midnight, may be summarised by the statement that seamen are interested in every measure that shall promote uniformity in the manner in which time is kept at sea, and shall assist in removing the anomalies of existing usage.

2. The double use of civil time, for all matters affecting the internal life of a vessel, and of astronomical time for all data connected with navigational observations, had become so confirmed by tradition, that, until recently, the inconveniences to which it gave rise were accepted by seamen without complaint.

3. Recent events, which will now be described, made the anomalies of such practice more evident, and seamen began to express a wish for reform.

4. The events to which reference has been made, and the consequences arising from them were as follows:—

- (i) The *de facto* employment in H.M. Fleet of a civil day of from 0 hours to 24 hours, beginning at midnight, for all telegrams and messages. The adoption of this system led seamen to inquire whether it could not be extended in such a way that all events on board should be registered by the same system of time notation.
- (ii) The decision to adopt a civil day of from 0 hours to 24 hours in the Admiralty tide tables for 1920.

When this intention was made known to British seamen, who had for long desired the change, they were naturally led to inquire whether the new system of time notation might not be still further extended, and made to embrace the astronomical data given in the tide tables as well. But, as this latter information is obtained entirely from the *Nautical Almanac*, it is difficult to see how the wishes of seamen can be met, unless the alteration be extended to that book also.

- (iii) The decision of the French Hydrographic Department to refer the astronomical data in the Abridged French Nautical Almanac for the use of seamen to the hours of a civil day of from 0 hours to 24 hours beginning at midnight.

This step can only have as consequence a decline in the sale of the British Almanac to seamen, if it shall continue to be drawn up on its present lines; for all navigators will prefer a publication arranged on the new method. Should the French Government take any steps to push the sale of their new Abridged Nautical Almanac amongst seamen, there can be very little doubt that such an effort would meet with success.

5. It now remains to describe the errors and inconveniences which the existing system may cause.

The Director of Navigation has expressed his views on one aspect of the subject as follows, and I entirely concur in them:—

“My personal opinion on this subject is, that it is contrary to

common sense to have two times in use in a ship, both from 0 hours to 24 hours, and yet 12 hours different.

"As an example, an officer takes a sight at 0600 on April 1, and yet has to look out his elements in the *Nautical Almanac* under the date of March 31; this may very easily lead to error, and probably does."

It has already been stated that the Admiralty tide tables will be brought into partial agreement with the new notation; but it should be remembered that seamen frequently consult local tide tables drawn up by independent harbour authorities as well. The data in these tables are based upon civil time; corresponding astronomical data have to be taken from the *Nautical Almanac*; for the tables referred to omit all information as to moon's transit, lunar declination, etc. The two sources of information are, therefore, arranged upon different systems of time notation; and the co-relation of astronomical and tidal data will always be attended by difficulties which open the way to errors, unless the two systems of time notation be made identical.

6. It should further be stated that the long acquiescence of seamen to the present system does not constitute an argument in its defence; for, previous to its adoption, they were involved in another even worse timekeeping anomaly.

7. In the preface to Captain J. L. Wharton's work, *Captain Cook's Journal*, 1893, p. xii, the following remarks upon the system of timekeeping will be found:—

"It was the usual custom on board ships to keep what was known as ship's time, *i.e.* the day began at noon *before* the civil reckoning, in which the day commences at midnight. Thus, whilst January 1, as ordinarily reckoned, is from midnight to midnight, in ship time it began at noon on December 31 and ended at noon on January 1, this period being called January 1. Hence the peculiarity all through the journal of the p.m. coming before the a.m. It results that any events recorded as occurring in the p.m. of January 1 in the log would, if translated into the ordinary system, be given as happening in the p.m. of December 31; while occurrences in the a.m. of January 1 would be equally in the a.m. of January 1 in both systems.

"This puzzling mode of keeping the day at sea continued to a late period, and was common to seamen of all nations."

The above passage will show that, although the confusion between civil and astronomical times was more marked in the past than it is now, the acceptance by seamen of the anomalies which remained when the present system of timekeeping at sea was introduced does not constitute a reason for their preservation.

8. It may be added in conclusion that, leaving out all questions of errors and mistakes, a mere enumeration of the contradictions arising out of the simultaneous employment of three separate systems of time notation on board—of the civil day of from 0 hours to 24 hours; of the civil day divided into two separate groups of 12 hours; and of the astronomical day from noon to

noon—would appear to constitute an almost irresistible argument in favour of uniformity. That uniformity is being achieved by the growing employment on board of a day of from 0 hours to 24 hours beginning at midnight; the necessity of reducing the data in the *Nautical Almanac* to that system has been already recognised by the French Government; and, from a seaman's standpoint, the advantages of such a change cannot be combated by any arguments in favour of the present arrangement.

Admiralty :
8th June, 1918.

*Descriptive Catalogue of a Collection of William Herschel Papers
presented to the Royal Astronomical Society by the late Sir
W. J. Herschel, Bart.*

(Drawn up by J. L. E. Dreyer.)

Shortly before his death (which took place on October 24, 1917), the late Sir William James Herschel, acting in consultation with his brother, Colonel J. Herschel, and with the consent of his son, offered to the Council of the Society as a free gift all such scientific manuscripts of his grandfather as the Council might wish to possess.

At the request of the Council I undertook to examine all the papers preserved at Slough. Many of them, such as musical compositions and original manuscripts of papers printed in the *Phil. Trans.*, were kept by the family, while all the rest, enumerated below, were handed over to the Society in January last.

It will be remembered that Sir John Herschel in 1863 presented to the Royal Society the seven original books of "sweeps" for nebulae, together with the "Register Sheets" of the 2500 Nebulae and Clusters and of Messier's objects, and also Caroline Herschel's Zone Catalogue.* Also that he in 1866 presented to the R.A.S. the similar register sheets of double stars (now bound in four volumes), and a copy of the sweeps in eight parts, written by Caroline Herschel. Thus all the original observation-books of William Herschel, and the systematically arranged copies made by his sister, are now preserved at Burlington House.

In 1910-11, when the *Scientific Papers of Sir William Herschel* were being published, the late Sir W. J. Herschel most liberally lent any papers asked for, which might be of use in connection with the new edition. But although I paid a visit to Slough in 1911, I had no opportunity of making a search myself among the manuscripts. It is only now that I have found, to my regret, several most interesting papers, of the existence of which I was hitherto unaware. The observations of variable stars should

* Cf. *Phil. Trans.*, 1864, p. 6. The register sheets of W. Herschel's nebulae are tied up in five packets, each between two wooden boards.

certainly have been printed in full in the new edition, while a more or less detailed account of the two treatises on the construction of specula might have been given in the Introduction.

The papers described in the following are all foolscap size except the last two. The "parts" were formed by a number of sheets being stitched together in a cover of limp grey cardboard. The original observations were in the early years written out by W. Herschel, afterwards always by his sister, who also transcribed the observations into separate fascicles according to subjects. The handwritings of the brother and sister were wonderfully alike.

References to volume and page are to the *Scientific Papers*, 1912.

Journal, 12 Parts.

Record of observations in chronological order. The part numbered XI. has apparently been lost.* The periods covered by the volumes are:

I. Mar. 1, 1774–Sept. 26, 1779	VII. Sept. 25, 1783–Feb. 19, 1784
II. July 15, 1780–May 14, 1781	VIII. Feb. 19, 1784–May 11, 1784
III. May 20, 1781–Jan. 20, 1782	IX. May 11, 1784–Sept. 7, 1784
IV. Feb. 6, 1782–Nov. 6, 1782	XA. Sept. 8, 1784–Dec. 21, 1784
V. Nov. 6, 1782–Apr. 2, 1783	XB. Jan. 11, 1787–Apr. 25, 1787
VI. Apr. 2, 1783–Sept. 24, 1783	XII. Mar. 7, 1794–Sept. 14, 1799

There is a remarkable gap between the first two parts, although many observations of double stars were made at that time.† The general appearance of a page of the earlier parts may be seen from Plate A (p. xxix) of vol. i. When the "sweeps" for nebulae began in 1783, the records of them were entered in the *Journal*, and beginning with Sweep 46 (December 18, 1783), also in the separate sweep-books now belonging to the Royal Society. But in XB. and XII. there are no records of sweeps, evidently because it was found useless to write the same thing in two places. XB. contains the discovery and early observations of the satellites of Uranus. XII. is chiefly a record of observations of planets; there are no observations of the relative brightness of stars. It is the last volume of the series, as there are seventeen blank leaves at the end.

Review, 8 Parts.

I. Aug. 17, 1779–Aug. 6, 1780	V. Feb. 14, 1792–Jan. 31, 1800
II. Aug. 19, 1780–Aug. 1781	VI. Feb. 4, 1800–Sept. 1, 1802
III. Aug. 1781–Nov. 1781	VII. Sept. 4, 1802–Sept. 25, 1810
IV. Jan. 1782–Aug. 1783	VIII. Sept. 30, 1810–July 4, 1819

* In her list of MSS., Caroline Herschel writes: "Journal in twelve volumes, No. 10 being in two books, A and B."

† They are given in the first part of the "Fixt Stars" series.

There is a great difference between the first four and the last four parts. On August 17, 1779, Herschel began his "second review of the heavens" with a 7-foot telescope of 6.2 inches aperture, power 227, which resulted in his first catalogue of double stars and the discovery of Uranus. The first three parts of this series contain lists of all the stars in the constellations from Harris' maps,* with marks opposite to them showing whether the stars on examination had been found to be single or double, what colour had been noted, and what telescopic stars in the immediate neighbourhood had been examined. Similarly, the fourth part deals with the "third review" of all Flamsteed's stars and small stars near them, using the same telescope with a power of 460. There are no measures of the double stars thus found, these being recorded in the *Journal*. Part 4 is on the cover stated to end with August 1783, but there are a few observations in the second paper on double stars from the following months down to January 23, 1784.

The last four parts contain, like the *Journal*, a chronologically arranged record of observations. In Part 5 are the observations of relative brightness of stars, and we find that these were not interrupted by Herschel's holiday trips. Thus, in August and September 1795, observations were made at Gloucester, Glastonbury, Salisbury, etc. Parts 5 and 6 contain the "Review of the Ecliptic," carried out on nineteen nights in 1792-95 and again during the last five months in 1801.† Evidently commenced in the hope of finding a new planet, this review in 1801 changed to a search for the lost Ceres. In Parts 6-7 are the observations of Binary Stars, and there are many observations of sunspots‡ and of Messier's Nebulæ and Clusters in this series. In Part 8 are the "Experiments on the relative distance of stars," published in 1817. The last observations recorded are of the comet of July 1819;§ it is merely noted that it was too near the horizon to allow high powers to be used (July 4-7).

Observations with the 40-foot Telescope.

A fasciculus, of which only the first eight pages have been used—the rest are blank. Contains records of the use of the telescope on seventeen nights, from October 24, 1788, to April 9, 1793.

The second and thicker 4-foot speculum was cast in February 1788, ground in July and September, and the polishing was commenced on October 1. The record of observations begins thus:—

"October 24, 1788.—I tried my second new speculum upon Saturn. The focal length I find is about 14 or 15 inches more than will be convenient. The polish is not brought up to the edges nor to the middle, as I intend to shorten the focus before I

* Joseph Harris: *Stellarum fixarum hemisphaerium boreale (australe)*. For the year 1690, 2 plates, 27 inch. square. London, s.a.

† See vol. ii. p. 614.

‡ Including some made in 1801 in various places in Wales.

§ See *Memoir of Caroline Herschel*, p. 131.

finish the speculum. I saw a very dark belt upon Saturn on that part of the disk where the ring crosses it. I am very well convinced that the edge of the ring consists of matter that reflects hardly any light; however, the approaching vanishing of the ring will confirm or refute this opinion much better. I saw also the 4th satellite *nf* and the 2nd and 3rd a few degrees *nf*. The speculum is much affected with lateral faults, but by drawing the eye sideways I saw very well.

"October 28, 1788.—By way of trying the motions I began to sweep. The P.D. piece, the speaking pipe, the hanging seat, and all the movements acted extremely well. The commodity of observing with this instrument is far beyond what I ever had before."

On August 28, 1789, the discovery of Enceladus was confirmed (see vol. i. p. lii), after which Saturn was viewed with the 40-foot on six nights up to December 2. On September 25, 1791, a "trial sweep for the ecliptic" was made with a power of about 370, and on September 28 a "first sweep of the ecliptic"; but nothing was recorded except transits of a few Flamsteed stars.

Fixed Stars, 7 Parts.

Seems to be nothing but a fair copy of the *Journal*, omitting the observations of sun, moon, and planets. This series covers the period from March 7, 1775, to July 12, 1784, without any interruptions. The pagination is carried on through the seven parts from p. 1 to p. 649, and there are twenty-nine blank pages at the end of Part 7. The first 236 sweeps for nebulae are included; and as there was nothing to enter from 1783 except sweeps and star-gauges, which were recorded in separate books, the series was discontinued in July 1784.

The original observations have not been copied literally, but a free paraphrase is given; for instance, in the sweeps the abbreviations pB, vS, mE, etc., are not used, but the descriptions are given in full: pretty bright, very small, much extended, etc. The discovery of Uranus is described thus:*

"N.B., about 10^h 30^m, . . . near 4/ in the quartile nearest to ζ Tauri, is a curious nebulous star. Perhaps it is a comet. The appearance of this star was so different from the rest that I examined it a good deal and compared it with other stars to see if my telescope was faulty, but found it in good order.

"11^h 30^m. Suspecting the star to be a comet, I took a superficial measure merely by comparing it to my field of view, and found that it was followed by a small star at the distance of two-thirds of the field, and that the small star seemed to follow it directly."

The new star not being a "fixt star," the observations of it after March 17 are not recorded in this series.

* Compare the facsimile of the original record (in the *Journal*) in vol. i., Plate A and p. xxix, and the account, p. 30, from the *Phil. Trans.*

*Observations on Changeable Stars.**Observations on Stars I suspect to be Changeable.*

A wrapper with twenty-eight loose leaves of observations collected and copied by Caroline Herschel from the *Journal, Review*, etc., four pages of "Remarks" on the nomenclature used in the catalogues of relative brightness, the methods of observation, etc. The second line of the title is in Herschel's handwriting. The stars are:

α Herculis.	Mayer 420 [R Leonis] (1795-96, not seen).
α Ursæ min.	
β Herculis.	β Persei 1779-83.
ζ Herculis.	— 1779-1804.
π Herculis.	β Lyrae.
$\circ$ Ceti (from 1777 to 1810).	η Aquilæ.
χ Cygni.	δ Cephei.
34 Cygni.	In the Crown [R Coronæ].
	South of 6 Aquilæ [R Scuti].

*Single Stars of the First Magnitude with the Nearest
Surrounding Small Stars.*

Eleven loose sheets in a wrapper, with neat drawings of fields with a bright star in the centre. The drawings were all made in 1783, but a few notes have been added later. Copied from the "Fixt Stars," the pages of that series (as well as the dates) being quoted.

*A Catalogue of the Stars which have been Observed by Wm. Herschel
in a Series of Sweeps, brought into Zones of N.P. Distance and
Order of R.A.*

A volume, half bound, in general appearance like Caroline Herschel's Zone Catalogue of Nebulæ, to which it forms a companion volume.* Gives places of the stars observed as reference stars in the sweeps, reduced to 1800 with the observed magnitude; first stars within 10° of the Pole in one list, then zones of one degree from 10° to 123° of N.P.D. The R.A. and N.P.D. are given to $0^{\text{h}}.1$ and $1''$ respectively. The last column is "intirely reserved for Wm. Herschel's discoveries," giving reference numbers to stars not in the catalogues of Flamsteed, Bode, or Wollaston.

Sun, Moon, Planets, Comets, 14 Parts.

(Extracted from the original Journal or Review.)

Sun and Mercury, 2 parts: first, 94 pp. (1779-1802); second, only 14 pp., rest blank (1802-18).

Venus, 1 part, 29 pp., rest blank (1777-1813).

* Alluded to in Caroline Herschel's *Memoir*, etc., pp. 183 and 186.

Moon, 1 part, 78 pp., 1775-1807, only 9 pp. dealing with observations made after 1782.

Mars, 1 part, 51 pp., 1777-1809, last 6 pp. after 1783.

Jupiter, 1 part, 40 pp., 1774-1811, last 11 pp after 1783.

Eclipses of Jupiter's Satellites, 4 pp., 1774-83, rest blank.

Saturn, 2 parts, 163 pp., March 17, 1774 to August 1815.

Georgian Planet and Satellites, 2 parts, 138 pp., rest blank. 1781-1810, only 2 pp. after 1799.

Comets and Asteroids, 3 parts. Comets, Part 1, pp. 1-38 (1781-1811). Asteroids, pp. 39-68, and Part 2, pp. 1-4 to Feb. 1814 (rest blank).* Comets, Part 3, 10 pp. (Nov. 1811-Jan. 21, 1812, and two observations of Olbers' comet, March 1815). Rest blank.

Drawings of Planets.

A small portfolio with five drawings, white on black ground. One, "path of the new star" (no date), two of Jupiter (no dates), two of Saturn, May 23, 1779, and June 19, 1780.

Experiments on the Construction of Specula, 4 Volumes, 3 Indices.

A chronological record of the work of casting, grinding, and polishing specula, from 1773 to 1818. In four stout parts or volumes stitched together at home, like all the rest. Was originally all written in W. Herschel's own hand, but as Caroline Herschel had copied the original first volume, the latter was in 1893 presented to the Oxford University Observatory, while the copy was left with the other three volumes.† There are many quotations from this record in the Introduction to *Herschel's Papers* (1912). In all, 2160 separate "experiments" or operations are recorded, distributed thus:

Vol. I.	472 experiments up to Nov. 28, 1790.
„ II.	530 „ „ Dec. 16, 1793.
„ III.	662 „ „ Jan. 4, 1804.
„ IV.	496 „ „ Dec. 5, 1818.

There are three separate indices. One, of "successful experiments," arranged according to focal length, from 40 ft. to 7 ft. Another, "Index to the articles," and a third one, ruled in columns but never written.

* Begins with the hunt for Ceres Sept. 16, Oct. 6, 9, Dec. 7, 1801. Also Jan. 1, 29, Feb. 3 (searched four hours), and Feb. 7, 1802, but no disc could be found. On Feb. 13 he found it, and says he had seen it Nov. 7 and 9 "and last night"; but there is no record of these observations.

† The volume at Oxford is far from being as complete as the copy now belonging to the Society. The following particulars are not found in it:—All operations previous to April 26, 1778 (vol. i. p. xxiv *et seq.*), all the work on the 30-foot in 1781 (p. xxvii), the Memorandum of 1785 (p. xlv), all the work on the 40-foot previous to June 23, 1787 (pp. xlvi-xlvii), and the casting of the second mirror (p. xlviii).

On the Construction of Specula.

A digest of the experience gained by the work recorded in the four volumes. 129 leaves, many having writing on both sides. Divided into 59 sections.

*Results of Experiments on the Construction of Mirrors
for Telescopes.*

A different and more elaborate treatise. 179 leaves, nearly all written on both sides.* Divided into 40 sections, each subdivided into articles; many sections of the first redaction are made into articles here. Many paragraphs have been copied unaltered, others have been expanded and new ones added. References to the experiments in the four volumes are added throughout. The text consists nearly altogether of short, concise paragraphs. The sections deal with the structure of mirrors (focal length, thickness, proportion of copper to tin), casting, construction of polishers, "gutters" in them, length of stroke, side-motion, speed, testing of mirrors, various imperfections, cleaning and preserving them, etc.

To this second redaction belong a set of 59 very neatly executed drawings in sepia, of tools and parts of machinery; made by Sir John Herschel.

There is also an index to the four volumes of experiments, sorted according to the sections of the second redaction. Written on small slips tied up in six bundles.

Letter Book.

Folio volume, bound in vellum; contains drafts or copies of about 190 letters on scientific subjects, written by Herschel in the years 1781 to 1817 to correspondents at home and abroad.

Letters addressed to W. Herschel.

A great number of letters (about 700), nearly all on scientific subjects (see vol. i. pp. lvi-lviii), arranged alphabetically, in separate wrappers for each letter of the alphabet and also for a few very frequent correspondents. There is not very much of interest in these letters; most of the foreign ones are merely written to acknowledge the receipt of publications or to order or inquire about telescopes.

Biographical.

"Memorandums from which an historical account of my life may be drawn."

A small quarto fasciculus of 60 pages, the last 24 of which are blank. Written about 1818. Most of the contents have been

* Marked on the cover by Sir John Herschel: "Most perfect copy."

printed in the Introduction to the *Scientific Papers*. The last entry is a note about the first attempt to use the 40-foot telescope on February 19, 1787 (see p. xlvii).

There are two Appendices to these autobiographical notes; a list of diplomas of membership of learned societies, and the original warrant signed by King George IV., dated September 21, 1820, granting to Herschel a pension of £400 a year. As the Act (quoted in the warrant) enabling the King to continue pensions and allowances paid by George III. specially directed that such pension should not exceed the amount paid immediately before the King's death, it follows that Herschel must, in addition to his salary of £200 a year, have continued up to 1820 to draw the allowance of £200 a year for the upkeep of the great telescope, granted him in 1787 (see vol. i. p. xlvii).

A List of Manuscripts, etc.

Twenty pages quarto in a stiff cover. Written by Caroline Herschel.* The list of MSS. fills nine pages. In addition to those described above and those now belonging to the Royal Society, it mentions Experiments on Light and Heat, Modifications of Light, Letters to Jacob Herschel, Bath papers and other unpublished papers (printed in vol. i.). Then follow (pp. 10-15) "Account of Telescopes," "Account of Mirrors," "Account of Polishing Machines" (all printed in vol. i. pp. lv-lvi). Finally (pp. 17-20), a list of musical compositions.

William Herschel's Observations of Variable Stars and Stars suspected of Variability.

[The following observations, which only lately were found among Herschel's papers, were collected and copied by Caroline Herschel from the original Journal, Review, etc. Those already printed in the *Scientific Papers* have been omitted here.—J. L. E. DREYER.]

Observations on changeable Stars.

Observations on Stars I suspect to be changeable.†

64 a Herculis.

[For observations from 1783 May 12 to 1796 May 27, see *Scientific Papers*, vol. i. pp. 556 and 560.]

96 May 30. 64 ; 27 Ophiuchi.	96 June 9. 64 ; 27 Oph.
May 31. 64 ; 27 Oph.	June 10. 27 Oph ; 64.
June 1. 64 ; 27 Oph.	June 11. 27 Oph ; 64.
June 4. 64 ; 27 Oph.	June 14. 64 ; 27 Oph.

* Another copy of this was used in 1911 for the edition of Herschel's Works (vol. i. p. lv).

† [The second line of the title is in W. Herschel's handwriting.]

- | | |
|--|--|
| <p>96 June 24. 64 γ 27 Oph.
 June 25. 27 Oph γ 64 or 27
 Oph compared to 64 ten
 times — — — , , , , , .
 July 6. 27 Oph — , 64.
 July 7. 27 Oph — , 64.
 July 10. 27 Oph — 64.
 July 11. 27 Oph — 64.
 July 16. 27 Oph ; 64.
 July 21. 64 . 27 Oph.
 Aug. 5. 64 . 27 Oph.
 Aug. 6. By many views
 64 , 27 Oph.
 Aug. 7. 27 Oph ; 64.
 Aug. 17. 64 ; 27 Oph.
 Aug. 18. 64 ; 27 Oph.
 Aug. 27. 27 Oph ; 64.
 Aug. 28. 27 Oph , 64.
 Sept. 2. 27 Oph ; 64.
 Sept. 21. 27 Oph — 64.
 Sept. 24. 64 — 27 Oph.</p> | <p>96 Sept. 30. 27 Oph ; 64.
 97 Mar. 26. 27 Oph — 64.
 27 Oph — , 64.
 Apr. 19. 27 Oph γ 64.
 Sept. 21. 64 — , 27 Oph.
 98 Sept. 17. 64 , 27 Oph.
 1804 June 10. α continues its
 regular changes of mag-
 nitude ; it is now a
 little smaller than κ 27
 Ophiuchi.
 July 21. α is not at its
 maximum of bright-
 ness ; it is less than 27
 Ophiuchi.
 July 30. α is less than 27
 Ophiuchi.
 1810 Sept. 25. α is less than 27
 (κ) Oph.
 Oct. 4. α is now larger
 than 27 (κ) Ophiuchi.</p> |
|--|--|

α Ursæ minoris Fl. 1.

- 83 Aug. 18. $\alpha\beta - \gamma$.
- 95 Feb. 14. β is larger than α but not much.
 Mar. 28. $\beta - \alpha - \gamma$ or nearly $\beta\alpha - \gamma$.
 Apr. 3. $\beta - \alpha - \gamma$, moon light.
 Sept. 11. 7 , 1 or 7 . 1 — 14 Draconis.
 Oct. 25. 1 , 7 but 1 is much higher than 7.
 At present it is 1 , 7 , γ Draconis.
 Oct. 26. $\alpha - \beta$ α appears uncommonly bright.
 Oct. 28. Polaris : β The pole star seems to be decreased, or
 β is increased. The place of the moon may possibly
 influence appearances.
 Oct. 30. α , β The night is not favourable.
 Very clear $\alpha - \beta$.
 Nov. 3. 1 — 7 7 seems to be less than not long ago.
 Nov. 7. 1 — 7.
 Nov. 28. α Ursæ maj γ Polaris γ β 11^h.
- 96 Jan. 8. 1 ; 7 1 appears small and I am now pretty certain
 that it is to be placed among the changeable stars.
 50 Ursæ maj — 1.
 Jan. 9. 1 — 7 5^h 30' The deception arising from twilight is
 very remarkable, for I can hardly suppose that 1
 should be periodical so as to agree with our
 24 hours.
 Jan. 28. 7 , 1 50 Ursæ maj — , 1 The evening remarkably
 fine.
 Jan. 31. 7 γ 1 1 appears to be small.

- 96 Feb. 1. 7^h 30' I have no doubt but that the pole star is now less than it used to be, compared to the idea I have of its former brightness $7 \frac{1}{2}$ 1. The weather however is not favourable at present.
7. 1 very clear. 1. 34 Aurigæ.
- Feb. 2. 7^h 50' 1 appears to be larger than last night, it is now 1; 7. The night very fine.
12^h It is now 7; 1. I ascribe the change however to the greater elevation of 7.
- Feb. 10. 7, 1.
- Mar. 27. 7-1 They are nearly at equal altitudes. The air is very clear.
- Mar. 28. 11^h 10' 7 $\supset$ 1 I suppose 7 to be larger than usual and 1 less than its ordinary magnitude. The air is changeable and I see them also $7 \frac{1}{2}$ 1. 50 Ursæ maj $\frac{1}{2}$ 7 and also $\supset$.
- Mar. 29. 9^h 15' 7-1 10^h 30' The night tho' apparently clear is not proper for this work.
50 Ursæ maj - 7-1 This upon the whole is the medium of these two hours appearance of these stars.
- April 3. 9^h 10' $7 \frac{1}{2}$ 1 It is also 7-1 very clear. They are nearly at equal altitudes. 7-1.
- April 4. $7 \frac{1}{2}$ 1 The result of many times observations. Sometimes 7? 1 but generally 7-1.
- April 5. 7-1 all the evening. 7 $\supset$ 1.
- April 9. 7-, 1.
- April 20. $7 \frac{1}{2}$ 1 The moon having been in the way this week past, I have however observed these two stars constantly and found 7 larger than 1.
- April 24. $7 \frac{1}{2}$ 1 The evening very clear.
- April 25. $7 \frac{1}{2}$ 1 The evening very clear.
- April 26. 7-1 The evening very fine. Possibly 1 may be a little affected by the greater brightness of the twilight and the neighbouring Zodiacal light * than 7 which is farther from it.
- May 4. 7, 1.
- May 11. $7 \frac{1}{2}$ 1.
- May 22. 7, 1.
- May 26. $7 \frac{1}{2}$ 1 I suppose 1 to be encreasing.
33 Draconis - 7. 33 Drac. - 1 - - 23 Draconis.
- May 30. 7, 1.
- June 1. 7, 1.
- June 4. 7; 1.
- June 9. 1. 7 or 1 $\supset$ 7 1 - - 5 Cephei.
77 Ursæ maj, 1, 79 Ursæ maj.
- June 10. 1 - - 5 Cephei. 1; 7.
- June 11. 1; 7.
- June 14. 1 $\frac{1}{2}$ 7 1 - - 5 Cephei. 77 Ursæ maj, 1 - 79 Ursæ maj.

* [*Query Aurora.*]

96 June 24. 1, 7 77 Ursæ maj; 1; 79 Ursæ majoris.

June 25. 1877. This is the result of 11 observations that were — — — — —, . . .

July 6. 1-7 1.77 Ursæ majoris.

July 10. 1-7.

July 11. 1-7.

July 21. 1-7.

Aug. 5. 7-1 It is 1-7.

Aug. 6. By many views
I, 7.

Aug. 7. 1 - 7.

Aug. 17. 1-7.

Aug. 18. 1-7.

Aug. 27. 1 - 7.

Aug. 28. 1-7.

Sept. 2. I - 7.

Sept. 8. I $\frac{1}{2}$ 7.

Sept. 21. I ; 7.

October 25. 1 —, 33 Draconis. 1 & 7 are both very bright, and γ Draconis seems to be less bright than usual 1, 7.

97 Jan. 26. 7; 1 or 7; 1 or 7 $\frac{1}{2}$ 1 7-1 the night is very fine
and both objects are steady & clear, many views
confirm it. It seems to be 1 that is changed.

Jan. 31. 1. 7.

Mar. 23. 7-1 1 seems to be less than usual 9^h , at 10^h at 11^h
the same.

Mar. 25. 1; 7 1 is evidently larger than it was March 23.

Mar. 28. 7-1.

Apr. 19. 7-1.

Apr. 24. 7—, I.

Apr. 25. 7- , 1.

June 5. 1-7.

July 15. 1 - 7.

98 Apr. 7. 7—, I.

98* Dec. 30. $1 = \frac{1}{7}$ but there is yet too much daylight.

9^h 1, 7 I cannot account for the change in the apparent magnitude of 1 except it should be owing to the deception of the twilight. The difference of the colour is perhaps also concerned; 7 being red. 11^h 1; 7.

27 β *Herculis*.

83 May 12. $\beta \zeta \alpha$.

95 May 18. 27—64—20 40—27.

May 22. 10^h 25' 40.27 or 27, 40 27-64 27.40.

May 23. 27-40 or at least 27, 40.

July 6. 27, 40 or 27. 40 but oftner 27, 40.

Sept. 16. 8^h 3' 27, 40 60 Ophiuchi. 27.

Sept. 17. $64 < 27$ or $27, 64$.

7^h 40' 27, 64-65 60 Oph, 27, 64 40.27 or at
most 27, 40 27.- or -, 67.

Sept. 18. 27-64, 65 40, 27 or 40. 27 I suspect 27 to be
changeable for it is now 60 Ophiuchi. 27 or even
27, 60 Oph. 27.40 or nearly 27, 40 27-67-65.

* [Should no doubt be 95, as there is a reference at foot of first page:
"See for Dec. 30, 95 after Ap^l 7, 98."]

- 95 Sept. 19. 60 Ophiuchi, 27.
 Sept. 20. 40 ; 27 27 - 60 Oph.
 Sept. 21. 40 = 27 23 Draconis -, 27, 12 Drac.
 40 ; 27 ; 60 Oph < 27.
 Sept. 22. 7^h 40 ; 27 27 - 60 Oph. 23 Drac -, 27, 12
 Drac. 27 - 65.
 Oct. 2. 40 ; 27. 23 Drac - 27 - 12 Drac. 60 Oph, 27.
 Oct. 9. 40 - 27 27, 60 Ophiuchi.
 Oct. 15. 40 ; 27.
 Oct. 25. 40, 27 27, 60 Ophiuchi.
 Oct. 26. 40, 27.
 Nov. 3. 40, 27 Twilight. 40, 27 ; 67 ; 65, 44 - 58.
 Nov. 6. 40, 27.
 Nov. 9. 40. 27.
 Nov. 20. 40' 27.
 Dec. 15. 5^h in the morning of the 16th 40, 27.
- 96 Apr. 4. 27 ; 40.
 Apr. 9. 27 - 40.
 May 19. 27 - 64 -, 27 Oph.
 June 1. 27 ; 40.
 June 9. 27, 40.
 June 10. 40 ; 27.
 June 11. 27 ; 40.
- 96 June 14. 40, 27.
 97 May 26. 24 Serpentis, 27.
- 40 ζ *Herculis*.
- 83 May 12. β ζ α .
 95 May 18. 40 - 27.
 May 22. 10^h 25' 40 . 27 or 27, 40 This evening the star 40
 seems to be less bright than it should be.
 37 Serpentis - 40 27 . 40 — 64, 65.
 May 23. 27 - 40 or at least 27, 40.
 July 6. 27, 40 or 27 . 40 but oftner 27, 40.
 Sept. 16. 27, 40 8^h 3' . 8^h 20' 40 is less bright than usual.
 The greater brightness of α was not temporary but
 continued all the evening. If it be a periodical
 change, the period is probably a pretty long one.
 ζ also seemed to remain smaller than usual but its
 change is not so considerable as that of α .
 Sept. 17. 7^h 20' ζ seems to be larger than last night. α π and
 ζ seem to be changeable. 7^h 40' 40 . 27 or at most
 27, 40 40 - 65 or -, 65.
 Sept. 18. 40, 27 or 40 . 27 27 . 40 or nearly 27, 40.
 Sept. 19. 40 - 67 - 65 But the air is bad and the observations
 become doubtful, though I have watched a great
 while for the above.
 Sept. 20. 40 - 65 β Draconis - 40, 12 Drac. 40 ; 27.
 Sept. 21. 40 = 27 23 Drac. - 40 - 12 Drac. 40 ; 27 ; 60
 Ophiuchi.

- 95 Sept. 22. 7^h 40 $\bar{7}$ 27 40 $\bar{7}$, 65 23 Drac $\bar{7}$ 40 $\bar{2}$ 12 Drac.
 Oct. 2. 40 $\bar{7}$ 27 40 $\bar{7}$, 65 23 Drac-40 $\bar{7}$, 12 Drac.
 Oct. 9. 40 $\bar{7}$ 27.
 Oct. 15. 40 $\bar{7}$ 27 40 $\bar{7}$ 65 23 Drac, 40 $\bar{7}$ 12 Drac.
 Oct. 25. 40 $\bar{7}$ 27.
 Oct. 26. 40 $\bar{7}$ 27.
 Nov. 3. 40 $\bar{7}$ 27 Twilight 40 $\bar{7}$ 27 $\bar{7}$ 67.
 Nov. 6. 40 $\bar{7}$ 27.
 Nov. 9. 40 $\bar{7}$ 27 40 $\bar{7}$ 67-65.
 Nov. 20. 40 $\bar{7}$ 27.
 Dec. 15. 5^h in the morning of the 16th 40 $\bar{7}$ 27.
 96 April 4. 27 $\bar{7}$ 40.
 April 9. 27 $\bar{7}$ 40.
 June 1. 27 $\bar{7}$ 40.
 June 9. 27 $\bar{7}$ 40 40 $\bar{7}$, 67 The latter appears small.
 June 10. 40 $\bar{7}$ 27 40 $\bar{7}$ 24 Serpentis. I suppose the observation
 May 22, 1795 should be 24 instead of 37 Serpentis,
 at present however the stars cannot be fairly com-
 pared together on account of the lowness of 24 &
 37 Serpentis. 23 Draconis, 40 $\bar{7}$ 22 Drac.
 June 11. 27 $\bar{7}$ 40 24 Serpentis-40.
 June 14. 40 $\bar{7}$ 27.
 June 24. 40 $\bar{7}$ - 67.
 Sept. 21. 40 $\bar{7}$ - 67 $\bar{7}$, 55.
 97 April 30. 40 $\bar{7}$ 27 Bootis.

67 π *Herculis.*

- 83 May 12. δ η π γ ϵ μ .
 95 May 22. 64 $\bar{7}$ 67 or 64 $\bar{7}$ 67, 65.
 July 6. 67 $\bar{7}$ 27 Ophiuchi, 64.
 Sept. 16. 64 $\bar{7}$ 67 67 I think was less bright this evening
 than usual.
 Sept. 17. 67 seems to be larger than last night.
 α π and ζ seem to be changeable.
 7^h 20' 67 $\bar{7}$ 65 or -65 27 $\bar{7}$ or - 67.
 Sept. 18. 27 $\bar{7}$ 67-65 67 seems brighter than last night.
 Sept. 19. 67 $\bar{7}$ 64 $\bar{7}$ 65 40 $\bar{7}$ 67-65 But the air is bad and
 the observations become doubtful, though I have
 watched a great while for the above.
 Sept. 20. 67 $\bar{7}$ 64 $\bar{7}$ 67-65 12 Drac, 67.
 Sept. 21. 67 $\bar{7}$ 65 67 $\bar{7}$ 12 Draconis or 12 Draconis, 67.
 Sept. 22. 67 $\bar{7}$ 65 67 $\bar{7}$ 12 Draconis.
 Oct. 2. 67 $\bar{7}$ 65 67 appears small this evening.
 Oct. 9. 67 $\bar{7}$ 44 67-64.
 Oct. 10. 67 $\bar{7}$ or - 44 67 $\bar{7}$ = 65 67 is very large.
 Oct. 11. 67 $\bar{7}$ 65 67 $\bar{7}$ - 44 67 seems less than last night.
 Oct. 15. 67 $\bar{7}$ 65 67 $\bar{7}$ 44 67 looks small.
 Oct. 16. 67 $\bar{7}$ 65 67 $\bar{7}$ 44.
 Oct. 25. 67 $\bar{7}$ 65 67 $\bar{7}$ - 44.

- 95 Nov. 3. $40, 27 \div 67 \div 65, 44 - 58$.
 Nov. 9. $40 \div 67 - 65$.
 Dec. 15. 5^h in the morning of the 16th $67 \div 65$.
 96 June 9. $40 - , 67$ The latter appears small. $67 \div 44$.
 June 10. $67 \div 65$ $67 = \div 44$ 22 Draconis, 67, 12 Drac.
 June 11. $67 \div 44$.
 June 14. $67 - , 44$.
 June 24. $67 \div 65$ $67 \div 44$ 67 looks small, 44 looks large.
 I compare each to itself by the idea I have of them. $40 - - 67$.
 Sept. 21. $40 - , 67 - , 55$.

o Ceti Fl. 68.

[The observations from 1777 Oct. 20 to 1780 Dec. 17 have been printed in Herschel's *Scientific Papers*, vol. i. p. civ and p. 1.]

- 81 July 21. o Ceti not visible to the naked eye; the constellation very low, but I saw δ very well.
 Aug. 17. o Ceti 27 2 Rev. $57 P^{ts} - 1\frac{1}{4} = 1' 47'' 54'''$ just measure. Is less than δ .
 Oct. 5. o Ceti barely visible to the naked eye. δ very bright, but the moon interfered.
 Oct. 22. o Ceti hardly visible though not far from the meridian. With 460 perfectly round and sharp without the least ray. Garnet.
 Oct. 28. o Ceti 2 Rev. $26\frac{1}{2} P^{ts} + 38 = 1' 52'' 37'''$.
 82 July 20. o Ceti larger than β .
 Aug. 11. o Ceti as bright as β , both are low.
 Aug. 25. o Ceti Posit. o Rev. $36\frac{1}{2} P^{ts} - 31 = 2^\circ 32'$.
 so close are the wires that it is difficult to take an exact measure unless they were finer. Dist. 3 Rev. $27\frac{3}{4}$
 $P^{ts} - 20 = 1' 54'' 36'''$ a full measure but pretty accurate. Moon light. The colour is only pr. or rw. and though pretty high much less than α .



- Aug. 30. o Ceti less than γ
 Sept. 4. Pale r. or r. at most.
 Sept. 27. o Ceti considerably less than δ . pr. or at most r.
 Sept. 30. Is barely visible with the naked eye by moon light.
 Oct. 13. o Ceti much less than 78 (ν).
 83 Jan. 2. o Ceti invisible to the naked eye, but I found it by the 66th.
 Jan. 13. Is much less than the 10th magnitude because I have sought the place where it must be through so accurately that no star of the 10th magnitude has escaped me.
 July 28. I see the periodical star with the naked eye = δ Ceti.
 July 30. o Ceti not quite = δ .

- 83 Aug. 2. $\circ$ Ceti 460 red.
 Aug. 21. In the telescope, 460, of a fine garnet colour as in the year 1780 Sept. 8th, much less than δ .
 Aug. 24. But just visible to the naked eye, equal to 75 Ceti.
- 84 Sept. 13 and Sept. 15. $\circ$ Ceti not visible.
 Sept. 20. A star about the place of $\circ$ Ceti, just visible to the naked eye. $\circ$ Ceti (taken in the sweep) 8m garnet but not deep..
- 86 Sept. 20. Now visible to the naked eye. 6m.
 89 Sept. 20. Visible to the naked eye. I suppose it to be 6m. or 6.7m.
 Oct. 15. About 6m.
 Oct. 18. 5m.
 Oct. 20. 4.5m.
 Nov. 9. $\circ$ Ceti = α Piscium or larger.
 Nov. 15. $\circ$ Ceti = γ .
- 90 Sept. 17. As large as γ nearly, larger than δ .
 Oct. 9. Larger than θ , much larger than γ , almost equal to α .
 Nov. 2. Much larger than γ . Almost but not quite equal to α .
 Dec. 2. Less than γ . A very little larger than δ . Of a deep garnet colour.
 Dec. 4. Very nearly equal to δ .
 Dec. 25. $\circ$ Ceti 6m, less than 53 (χ).
 Dec. 28. 7m. The night is very fine, otherwise it would hardly be visible.
- 91 Oct. 16. $\circ$ Ceti is of the magnitude of γ nearly, or between that of γ and δ .
 Nov. 25. $\circ$ Ceti is reduced again to the 7th magnitude.
- 92 Aug. 14. $\circ$ Ceti is larger than γ .
 Sept. 8. Is larger than γ but not so large as α .
 Sept. 15. Is larger than γ but less than α .
 Oct. 16. Is about the magnitude of δ .
 Oct. 18. Is still a little larger than δ .
- 93 Sept. 27. Is much less than δ .
- 95 Nov. 10. By 66 $\circ$ Ceti is not visible. By 71 not visible.
 Dec. 14. 68 is not visible in the finder.
 Dec. 30. 68 is not visible in my achromatic finder. I am very sure of the place.
- 96 Jan. 28. 68 is not visible in my finder.
- 98 Feb. 18. 68. 82.
 Dec. 10. 68—82 2^h 22'.
- 99 Oct. 26. Is much less than δ . γ — δ — $\circ$ Ceti.
 Nov. 16. Is brighter than δ , being visibly encreased since I saw it last. Its mag. is between γ & δ .
 Nov. 20. Is equal to γ .
 Nov. 27. Is considerably larger than δ .
 Dec. 19. Is encreased since my last observation of it.
 Dec. 27. Is rather diminished. It is not much larger than δ .

- 1800 Jan. 2c. Is less than δ . $\alpha - - \delta$.
 Jan. 25. Is diminished very much. A little brighter than 63.
 Oct. 24. Is remarkably bright. It is brighter than α . It is brighter than β Arietis.
 01 Sept. 7. α is larger than δ .
 Sept. 9. α is larger than γ .
 Oct. 2. α is considerably larger than γ .
 Oct. 12. α is less than γ but larger than δ . With 400 it is single, with 600 the same and well defined.
 Nov. 1. α is less than δ . It is not a double star.
 07 Dec. 6. This star has lately made its appearance, it is now a little less than δ Ceti.
 Dec. 22. This star is now full as bright as β Arietis; it is of a brilliant reddish colour.
 Dec. 30. α Ceti is nearly as large as α Arietis. Its lustre is very brilliant.
 08 Jan. 15. This star is now much brighter than α . It is nearly as bright as α Arietis & is of a brilliant reddish colour.
 09 Jan. 11. Is $= \delta$ Ceti $6^h 10'$.
 Jan. 12. Is less than δ .
 10 Sept. 25. Is less than δ .
 Oct. 4. Is nearly equal to δ .
 Oct. 30. α Ceti has decreased again, it is much less than δ .

*χ Cygni Fl. 17.**

- 95 Oct. 26. I cannot find the periodical star 1° sf 17 (χ). There are some small stars in the neighbourhood, but they are very much smaller than χ . 21 — — 17. 8.
 Nov. 15. Near $48'$ following 17 χ in a line from α Lyræ through χ is the star which I suppose to be the changeable one in the neck. 17 — that star.
 Dec. 14. The periodical star sf 17 is not visible.
 Dec. 15. $47'$ south of 17 (χ) is what I now suppose to be the periodical star; it is about $8m$; or 17 — periodical. I have before looked for it in the wrong place I suppose. In a line parallel from 6 to 13.
 Dec. 30. There is no star that will answer to the periodical one $47'$ south of 17 (χ) according to Dec. 15.

* [C. H. describes this as "south for χ Cygni Fl. 17." It is χ Cygni, but 17 Cygni is $51'$ north of it.]

Fl. 34 Cygni.

- 92 Sept. 15. 12^h common time. The periodical star in the Swan's neck 34 Fl. is of between the 6th & 5th magnitude.
- Oct. 18. 34 Cygni is a little smaller than it was estimated to be on the 15th of Sept. It is barely as large as the 36 and much less than the 29th. The 29 is full as large as the 27th and 28th.
- 95 Nov. 9. 28. 34.
 Dec. 14. 34. 29.
 Dec. 30. 34-40 34. 29 34-36 28, 34.

30 *Hydræ Hevelii.*

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420 of Mayer [R Leonis].

- 95 Feb. 14. 420 of Mayer, the last of 3 in Bode's map, is not visible in my finder; nor in the 7 feet with the double eyeglass, unless the map is faulty in the representation, which I am inclined to believe.
- If the following of three is 420 then it is a little larger than 419. It is the most south and following of three that make an angle of 145 degrees. The two most south are near each other, the most north is near double the distance. The middle star is the least and the most south; or 420 is rather larger than the most north, but not much; or it may be said to be nearly alike.
- 95 Apr. 21. 18 & 19 Leonis are visible but not the 420 of Mayer which is said to be changeable.
- May 6 and May 8. 420 of Mayer not visible.
- 96 Feb. 1. Is not visible unless the first of three in a line should be it; which is a very small star of the size of the other two of that line which is parallel to α η Leonis.

β *Persei Fl. 26.*

[For observations up to and including 1783 May 3, see *Scientific Papers*, vol. i. p. cvii.]

- 83 May. 4. 9^h 15' Has its full light.
 15^h 0' Is very bright, ρ scarce visible.
 May 5. 9^h 30' Bright.
 May 6. 8^h 48' Has all its light.
 May 12. 9^h Of its usual magnitude, but too low for accuracy.
 May 20. 14^h 17' A very little brighter than ρ .

- 83 May 20. 14^h 30' A little brighter than ρ , but daylight is too strong, I can only estimate very imperfectly, however, the star is evidently under an occultation.
- May 21. 12^h 30', 13^h, 14^h as usual, at 14^h very bright and ρ very clear.
- May 25. 12^h 15' As usual.
- May 30. 13^h 5' Algol is not much brighter than ρ but there are flying clouds—very changeable—Algol bright— ρ not visible. 13^h 50' very bright.
- June 6. 12^h 40' Bright, ρ as usual. 13^h 40' has all its light, rw with 227 but too low for colours.
- July 23. Algol is very near as white as α Andromedæ.
- July 28. 9^h 26' Not visible to the naked eye tho' high enough to be seen if it were eclipsed.
- 9^d 40' In a night glass considerably larger than ρ .
- 10^h 0' I take it now to be a small star of the 3^d mag.
- 11^h 0' = γ Persei naked eye.
- 11^h 20' = γ Persei.
- 12^h 30' larger than γ Persei, nearly = β Arietis. With 460 of a fine bluish white colour.
- 14^h 0' seems only now or but lately to have regained all its light.
- Algol has a star about 70° sp of the 5th or 6th class excessively unequal, only seen after long looking.
- July 30. Algol in magnitude between α and β Arietis.
- Aug. 17. 9^h 50' Algol but very little brighter than ρ .
- 10 35 Decreased nearly to ρ .
- 10 55 Doubtful whether it is larger than ρ .
- 11 25 Equal to ρ ; certainly not larger. Both stars are round and well defined in the telescope, Algol bw, ρ red or pr.
- 12^h 0' Possibly a very little larger than ρ .
- 15 15 Intirely recovered its usual lustre.
- Aug. 20. 9^h 25' But little brighter than ρ .
- 9 55 A good deal recovered.
- Sept. 6. 7^h 35' Of the usual size, tho' rather too near the horizon to judge perfectly of its magnitude. I see it very plainly tho' ρ is scarcely visible.
- 8^h 5' of its usual size, larger than ρ , the colour is rw, but the star is affected by the prismatic power of the atmosphere.
- 8^h 45' As usual, w, perhaps a little inclining to red.
- 9^h 7' As usual. 9^h 15' perhaps a little diminished.
- 9^h 28' near = γ . 9 47 not much diminished.
- 9^h 55' nearly = γ or rather still larger.
- 10^h 0' Still larger than γ , the air is changeable and I can hardly be sure whether it is at all diminished or not.
- 10^h 10' Perhaps a little diminished.

- 83 Sept. 9. $7^h 40'$ Is perhaps a little diminished. Less than γ , but so much nearer the horizon that it is doubtful. Appears = ϵ and seems not to be in any cloudy atmosphere.
 $8^h 0'$ = ϵ visibly reduced, much larger than ρ .
 $8^h 30'$ still = ϵ , not less.
 $9^h 30'$ less than ϵ ; not much larger than ρ .
 $10 30$ less than ϵ , very little larger than ρ .
 $11 0$ = ϵ , larger than ρ .
- Sept. 12. $7^h 21'$ Not much larger than ρ , rather brighter than ϵ , less than γ .
 $9 10$ Full = γ or rather brighter.
 $10 6$ Larger than γ ; very bright.
- Oct. 2. $7^h 50'$ = ρ . I observed it $20'$ ago = ρ .
- 84 Sept. 7. $22^h 57'$ Sid. T. Not much larger than ρ . $23^h 18'$ a little larger than ρ .
- Sept. 10. $22^h 27'$ Algol but little larger than ρ Persei.
 $24^h 44'$ Of its usual brightness.
- 85 Sept. 12. $19^h 49'$ A very little less than γ Persei but larger than δ .
- Sept. 15. $19^h 8'$ A very little brighter than ρ Persei.
 $22 0$ Larger than γ .
- 87 Aug. 7. A little brighter than ρ Persei.
- 89 Sept. 20. $0^h 17'$ Algol very bright.
- 90 Dec. 2. $0^h 33'$:: One magnitude larger than ρ Persei. Larger than δ .
- 91 Sept. 26. Algol was eclipsed this evening; when it was most reduced it was still brighter than ρ Persei.
- 92 Oct. 23. $20^h 34'$ Algol is only a very little larger than ρ .
 $21^h 16'$ Is one magnitude larger than ρ Persei.
- 93 Aug. 20. $19^h 4'$:: Algol is but a very little brighter than ρ .
- 95 Mar. 22. The small star sp Algol of its usual lustre. Schröter calls it changeable. I have always seen it as I do this evening. (See J.B. 1791 page 219 where Schröter says he discovered it Oct 12, 1787 &c: See also J.B. 1796 p. 193 to 196.)
- Mar. 30. The star near Algol on the discovery of which Mr Schröter values himself so much was observed (page 396 Fixt Stars) July 28, 1783, and its place and distance pointed out in a coarse way but quite sufficient to identify it. It is not periodical or changeable, as far as I can find at present; for whenever I have looked for it I have always seen it and another. Even now by moon light and when the star is low I have seen it with 460 though not without some difficulty.
- Aug. 4. β Persei = γ Persei.
- Aug. 21. $11^h 25'$ common time not accurate. Algol not much larger than ρ .
 $12^h 0'$ nearly as small as ρ .

95 Oct. 16.



Position of two small stars near Algol. I tried to measure the angle, but with the power that is on the micrometer I could not see the small star, and it would not

bear illumination. By the figure it is about 78° sp.* The farthest is 64° sp. These estimations cannot be very accurate. 287. 460.

97 July 15. $18^h 55'$ Sid. Time. β is very faint. $\beta - \rho$ Persei.Sept. 22. $19^h 30'$ Sid. Time. Algol is a very little brighter than 23γ Persei. Cor. $- 3' 13''$. 1 $20^h 11'$ Algol has recovered much of its brightness.98 Dec. 10. $2^h 22'$ Andromeda 57 — — 26 Persei.99 Sept. 23. $9^h 40'$ common time. Algol is not much larger than ρ .1802 Feb. 3. $10^h 15'$ mean time. The star is eclipsed.

1803 Sept. 11. Early in the evening the star began to lose its light.

Sept. 14. As soon as it was dark enough to see the star I found it reduced to nearly the brightness of ρ Persei. $8^h 15'$ Algol is regaining its lost brightness but is still considerably fainter than usual. 10^h It has nearly regained &c.1804 Oct. 8. 7^h Algol is under an occultation. *β Lyræ Fl. 10.*82 May 12. To the naked eye γ much larger than β .85 Feb. 8. $15^h 7'$ β Lyræ a very little larger than γ .Mar. 5. $14^h 33'$ $\beta = \gamma$ or perhaps a little less. $15 50$ a very little less than γ .Mar. 10. $14^h 31'$ Much less than γ and evidently decreased from its usual size. $16 25$ $\beta = o$ Herculis.Mar. 13. $16^h 17'$ $\beta = \gamma$ perhaps a very little larger; much larger than o Herculis.Mar. 16. $15 29$ β a little less than γ , and a little larger than ζ , but nearer to γ than ζ .April 10. $17^h 0'$ β Lyræ much less than γ .Apr. 11. $16^h 56'$ β a little less than γ but considerably larger than last night about this time. Equal to $103 (o)$ Herculis.Apr. 14. $17^h 1'$ β Lyræ $= \gamma$.Apr. 26. $13^h 1'$ β larger than γ .Apr. 27. $14^h 1'$ β full equal to γ , but I think not so large as yesterday.Apr. 29. $14^h 45'$ β much smaller than γ .* [*ab* is the parallel.]

- 85 May 3. $14^h 8'$ β nearly equal to γ .
 May 5. $15^h 45'$ β a very little less than γ , about the size of 104 Herculis.
 May 13. $12^h 0'$ Very small, seems less than I have seen it before, at least a magnitude inferior to γ .
 May 14. $16^h 40'$ is considerably encreased since yesterday.
 May 21. $13^h 48'$ $\beta = \gamma$.
 May 22. $14^h 55'$ $\beta = \gamma$, perhaps a very little less; the colour of β is reddish, γ white.
 May 23. $14 45$ β nearly $= \gamma$.
 July 7. $16 46$ β nearly $= \gamma$.
 Sept. 6. $20 11$ β very small.
 Sept. 7. $21 36$ a little smaller than γ .
 Sept. 28. $0 59$ β less than γ .
 89 Sept. 20. $0 17$ β less than γ .
 95 May 5. $10 45$ common Time. β Lyræ much less than γ .
 Sept. 15. $11 0$ 10 seems to be at its minimum.
 96 June 1. $14 \frac{1}{2} 10$.
 June 9. $14, 10$.
 June 10. $14, 10$.
 June 25. $14 - - - 10$ I never saw 10 so small, it was with the naked eye $\zeta \frac{1}{2} \beta$ By β I mean the joint lustre of 6 and 7, as it appears to the eye.
 Sept. 2. $10 . 14$.
 Sept. 21. $10 \frac{1}{2} 14$ or $10 . 14$ The air is very undulating.

η Aquilæ Fl. 55.

- 85 Feb. 8. $15^h 7' 14''$ η very nearly equal to 41 (ι) but the situation is low.
 Mar. 10. $16 24 52$ $\eta = \iota$.
 Mar. 13. $16 17 18$ η is much encreased, being considerably larger than ι .
 April 14. $17 0 41$ η as small as ι or rather less.
 Aug. 12. $\eta 3 . 4$ m. It seems brighter than I have lately seen it.
 Sept. 7. $20^h 7' 55''$ $\eta = \iota$.
 96 June 1. $65 - , 55 \quad 55 , 41$.
 June 4. $41 ; 55$.
 June 9. $65 - 55 - 41$.
 97 Sept. 21. $41 ; 55$.

δ Cephei Fl. 27.

- 85 July 29. (Loompit Hill, Sir J. Banks told me of δ Cephei &c).
 $20^h 5'$ δ Cephei a very little less than ϵ .
 July 30. $20^h 20'$ Much larger than ϵ .
 Sept. 6. $20 11$ δ as large as $::$ [In pencil is added: "I believe it was compared to β Lyræ. See Sweep."]
 Sept. 7. $21 36$ δ Cephei a little less.
 Sept. 28. $0 59$ δ Cephei a little less than ζ .

96 June 1.	27.23	21 = $\frac{2}{5}$ 27.
June 9.	21 - 27 - 23.	
97 Mar. 28.	27 - 23.	
Apr. 19.	27.23.	

Mr. Pigott's south of 6 Aquilæ [R Scuti].

- 96 Oct. 25. Mr. Pigott's variable star south of 6 Aquilæ compared to the largest of two more towards the cluster of stars is P - α (calling the Periodical star P, the other α).
 Nov. 3. P - , α .

Mr. Pigott's in the Crown [R Coronæ].

- 96 Oct. 25. Mr. Pigott's variable star in the Crown from 13 towards 4 Coronæ. Its magnitude to another about 20' more towards 4 is P - , α (calling the Periodical star P and the other α).

[The abbreviations used in these observations are (*Scient. Papers*, i. p. 543):—

Simple Characters.

- ' The least perceptible difference less bright.
- . Equality.
- , The least perceptible difference more bright.
- A very small difference more bright.
- , A small difference more bright.
- - A considerable difference more bright.
- - - Any great difference more bright in general.

Compound Characters, expressing the wavering of Star-light.

- ∴ From the least perceptible difference less bright to equality.
- ; From equality to the least perceptible difference more bright.
- ∴ From a very small difference more bright, to the least perceptible difference.
- ⇒ From - , to - &c.
- ∴ The wavering expressed by the passing of the light from a state of the least perceptible difference less bright to equality, and to the least perceptible difference more bright.
- ∴ The wavering expressed by the changes from - to , and to . or from . to , and to - .

General Characters.

- = Perfect equality.
- < Less, but undetermined.
- > Larger, but undetermined.]

Observation of Nova Aquilæ on June 8.

By A. Grace Cook.

The Nova was observed at Stowmarket on June 8, while watching for meteors. Observations commenced at 9^h 30^m G.M.T. in the hope of seeing a bright meteor or fireball in the twilight sky. The stars that are then visible were noted for reference, and almost at once a bright star was noticed which had just appeared from behind a group of low trees. The constellation of Aquila was hidden by them, and on moving to where the region could be seen, it was at once apparent that the star equalled Altair in magnitude and appearance, except that the Nova was bluer and twinkled very much. Its position was marked on a star-chart approximately, and at 9^h 50^m G.M.T. the observation for meteors was continued till 11^h 45^m G.M.T.; the first meteor being recorded at 10^h 2^m.

After 11^h 45^m a naked-eye sketch and an opera-glass sketch were made of the stars in the neighbourhood of the Nova, which was situated on the northern edge of a rich branch of the Milky Way reaching to Scutum. A star followed the Nova by about 10 minutes of arc, which was found to be No. 3 in *Backhouse's Catalogue*, J.L. 35,005, Mag. 6.4, R.A. 18^h 44^m.5, +0° 43'. (Position, 1900.)

On the morning of June 9 a telegram was sent to Greenwich Observatory announcing the discovery:—"Nova, last night, first magnitude eighteen forty-five plus one. Grace Cook, Stowmarket."

At 9^h 30^m G.M.T. on June 9 the sky was very clear between clouds and the Nova showed up much brighter, rivalling Vega. It seemed to increase in brilliancy as the sky darkened, and by 11^h 15^m was a wonderful object, looking like Venus at her brightest,* and should have been visible in daylight. It was still twinkling a good deal.

On June 10 the Nova was seen for a few minutes before 10^h G.M.T., and was equal to Vega in brightness.

June 11 was cloudy and the Nova was dimly seen and seemed equal to Altair in magnitude.

1918 June 12.

* I undoubtedly overestimated the brightness of Nova Aquilæ on June 9. It was due, I believe, to the piercing, scintillating, white light of the Nova.—A. G. C.

Observations of Nova Aquilæ. By W. F. Denning.

On commencing a watch for meteors on June 8, I immediately observed that a new star of considerable brilliancy had made its appearance in the western border of Aquila. I have made the following rough estimations of its magnitude by comparisons with Altair and Vega (adopted magnitudes 0.74 and 0.10):—

				Mag.	Notes.
1918 June	8 ^d	10 ^h	0 ^m G.M.T.	0.9	Slightly < Altair.
	8	14	30 „	0.8	Very slightly < Altair.
	10	10	0 „	0.1	Much > Altair; not quite = Vega, but allowing for difference of altitude the brightness was con- sidered equal.
	11	10	0 „	0.35	Slightly < Vega. Much > Altair.
	11	12	15 „	0.45	Decidedly < Vega.

The night of June 9 was cloudy throughout.

I could not resist the impression, while intently watching the star, that it exhibited variations in brightness at short intervals. For periods of five or ten seconds it appeared to intensify and sparkle in a very pronounced manner after intervening lulls of two or three minutes. The passage of thin cloud or mist across the object might well have occasioned this effect, though at times the sky seemed perfectly clear during its occurrence.

The star showed up well in the strong twilight, and had observations been taken in the early evening only, the magnitudes from comparison with Altair would have been greater than those given.

On the previous night, June 7, I was engaged in meteoric work between 10^h 15^m and 12^h 45^m G.M.T., but noticed no strange object, and so conclude that the new star must have been faint on that night. If, however, it was as bright as 4th mag., it might have been missed, for my watch was generally directed to high altitudes, and I saw no meteors in the immediate region of Aquila. On June 4, at 11^h 21^m, I recorded a fine meteor, equal to Jupiter, commencing its flight ($278^{\circ} + 2\frac{1}{2}^{\circ}$ to $246^{\circ} - 19\frac{1}{2}^{\circ}$) very close to the position of the new star, but I remarked no unusual object there at that time.

Bristol:
1918 June 13.

Observations of the new Star in Aquila. By W. H. Steavenson.

About six years ago, shortly after the appearance of Nova Geminorum No. 2, I formed the habit of making a rough naked-eye survey of the Milky Way every fine night. On 1918 June 8 I began my survey at 10.30 G.M.T., and immediately noticed what was obviously a Nova between δ Aquilæ and η Serpentis. The approximate position of the object was determined from a star Atlas, and communicated by telephone to the Royal Observatory at 11.10, the delay being due to the considerable distance of the nearest public call-office.

On returning home at 12.0 I observed the Nova in the 15-inch reflector. The colour appeared to be pure white, with perhaps a very faint tinge of blue. On comparing it with Altair it was practically impossible to detect any difference of colour in the two stars. The spectrum of the Nova, as seen with a small direct vision spectroscope, was, so far as I could see, continuous, with no certain evidence of either dark or bright lines. The following magnitude estimations were made with the naked eye, the comparison star being α Aquilæ, whose magnitude is 0.9:—

G.M.T.	Comparison.	Deduced Mag.
h m		
10 30	.2 < Altair	1.1
12 0	.1 < Altair	1.0
13 0	? = Altair	0.9
14 0	= Altair	0.9

Two photographs were taken at 13^h 10^m and 13^h 20^m respectively. The camera used was fitted with a lantern lens of 7.4-inch focus, working at $f/3.5$. It was not possible to mount the apparatus properly, so it was merely tilted up on the flat roof of the house and allowed to remain stationary. In each case the Nova and Altair were both allowed to make short trails on the plate with a view to comparing their respective photographic magnitudes. Examination of the plates seems to indicate that the Nova was, at 13^h 20^m, about .2 mag. fainter than Altair, but the trail of the latter is probably broader than it should be, owing to a slight error in the centering of the lens.

On the following evening, June 9, the Nova was first observed at 9^h 0^m, when it was at once obvious that its brightness had very greatly increased. It was somewhat low in the sky, but the latter was extremely transparent, so that a good estimate of magnitude, using Vega and Altair as comparison stars, could be made. My estimates at 9.45 were as follows:—

Comparison.	Deduced Mag.	Mean.
0.5 > Vega (Mag. = 0.1)	-0.4 }	-0.5
1.5 > Altair	-0.6 }	

By 11.30 the sky was rather more uniformly dark, so that Vega was no longer seen against a relatively pale sky, as was the case at 9.45. The result was that this star appeared brighter than before, as compared with Altair. This no doubt accounts for the apparent discrepancy between the estimates made at the two times, according to the comparison star employed. My estimates for 11.30 were:—

Comparison.	Deduced Mag.	Mean.
0.5 > Vega	-0.4	-0.62
1.75 > Altair	-0.85	

On June 10 there was a good deal of cloud about, and the sky was also somewhat hazy at low altitudes. This made comparisons of the Nova with other stars somewhat difficult, and the following figures must be considered as being rather uncertain. Time—10.30 G.M.T.

Comparison.	Deduced Mag.	Mean.
Nova = Vega	0.1	0.0
1.0 > Altair	-0.1	

The Nova appeared to me to be somewhat yellowish. This may have been partly an atmospheric effect, but my impression was that Altair, which was at much the same altitude, was not so yellow.

On June 11 the sky became fairly clear late in the evening, and the following estimates were made at 11.15:—

Comparison.	Deduced Mag.	Mean.
0.5 < Vega	0.6	0.5
0.5 > Altair	0.4	
$\frac{1}{2}$ Vega - Altair	0.5	

The Nova seemed in the 15-inch to be of a very pale pinkish-orange tint, and the small chemical spectroscope, with slit removed, showed a strong bright line in the extreme red (presumably H_{α}) and another in the blue (H_{β} ?).

On June 12, at 11.15, my estimates were:—

Comparison.	Deduced Mag.	Mean.
0.5 < Vega	0.6	0.7
0.1 > Altair	0.8	

On June 13, in a very transparent sky at 10.15, I made the Nova 0.1 fainter than Altair. Deduced mag. = 1.0.

West Norwood:
1918 June 14.

Observations of Nova Aquilæ (1918). By Felix de Roy.*(Communicated by Professor Turner.)*

Nova Aquilæ, No. 3, was independently discovered here on June 8, 10^h 45^m G.M.T., when pointing R Scuti in a pair of binoculars, and up to the present day has been seen on each night since.

Its light has been compared with the following stars, whose numbers, magnitudes, and types of spectra are extracted from the Revised Harvard Photometry (*Annals H.C.O.*, 50):—

Description.	No.	Name.	Mag.	Spectrum.
A	7001	α Lyræ	0.14	A
B	7557	α Aquilæ	0.89	A 5
C	7924	α Cygni	1.33	A 2

Owing to the large influence of the atmospheric absorption in observations of this kind, where the Nova has been compared with very distant stars in unequal altitudes, the "step" method has been entirely discarded and the fractional method used, in which the observer simply tries to establish a simple arithmetical proportion between the apparent brightness of the compared objects.

For the same reason the observations have all been reduced to the zenith. It seems obvious that estimates of this kind, especially those made at small altitudes, cannot be correctly reduced by simply working up the magnitudes of the comparison stars as they occur in the catalogues where they are reduced to the zenith. α Aquilæ, for example, which is rated 0.9 in the Revised Harvard Photometry, never appears brighter than about 1.0 under our latitudes, whereas α Lyræ can acquire its full catalogue value (0.1).

In the reduction of the following observations, the catalogue magnitudes of the comparison stars have been reduced to their apparent magnitude at the moment of the estimate, according to their zenithal distance; the fractional method has then been applied, and the resulting apparent brightness of the Nova has been brought up to the photometric scale by adding the corresponding value of the absorption.

1918.	G.M.T.		Estimate.	Zenithal Distance of Nova.	Magnitude.		Weight. (1-3).	Remarks.
	h	m			Apparent.	Corrected.		
June 8	10	45	B 3 V 1 C	63	1.37	1.09	3	
	12	25	B 3 V 1 C	53	1.28	1.13	3	
	14	2	B 1 V 1 C	51	1.15	1.02	2	Twilight.
						1.08		
9	9	35	V 2/7 $\overline{A-B} > A$	71	-0.16	-0.64	2	Twilight.
	10	20	V 3/7 $\overline{A-B} > A$	65	-0.25	-0.57	2	
	11	40	V 5/7 $\overline{A-B} > A$	56	-0.45	-0.63	2	Overcast afterwards.
						-0.61		
10	9	35	A 3 V 4 B	71	0.74	0.26	2	Twilight.
	11	23	A 3 V 4 B	58	0.56	0.36	3	
	13	10	A 3 V 4 B	51	0.50	0.37	3	
	13	45	A 1 V 1 B	51	0.56	0.43	2	Twilight.
						0.36		
11	10	33	A 2 V 1 B	63	0.85	0.57	1	Clouds.
	11	10	A 5 V 4 B	58	0.69	0.49	2	
	12	9	A 5 V 3 B	54	0.69	0.53	3	Good.
						0.53		
12	9	42	A 5 V 2 B	69	1.03	0.61	2	Clouds, twilight.
	11	1	A 5 V 3 B	59	0.76	0.54	3	Good.
	13	14	A 3 V 1 B	51	0.77	0.64	3	
						0.60		
13	10	5	B 1 V 5 C	65	1.28	0.96	2	Twilight.
	12	8	B 1 V 2 C	53	1.13	0.98	2	Clouds.
						0.97		

The extrapolation on June 9 was difficult, and, in the light of the experience at Harvard College in comparing naked-eye estimates and photometric measures of Nova Persei, No. 2, may have given underestimated values.

The colour of the Nova has been estimated as follows in a 3½-inch refractor, power 30:—

- June 8. Pure brilliant white.
 9. Bluish-white, distinctly bluer than Vega.
 10. Brilliant white, tinge of yellow just suspected.
 11. Still white, but distinct yellowish tinge.
 12. Slightly, but distinctly yellowish.
 13. Clearly yellowish.

Thornton Heath, Surrey:
 1918 June 14.

Nova Aquilæ (1918). By Harold Thomson.

The Nova was discovered at 11.44 G.M.T., 1918 June 8, and the Royal Observatory notified by telephone. At the time of discovery the Nova was very white, and about equal in magnitude to Altair.

The spectrum was examined frequently during the night with a Hilger-Zöllner eyepiece spectroscope attached to the 12½-inch reflector.

There were no *prominent* lines in the spectrum, either bright or dark, but several lines of each kind were suspected, including two very faint bright lines in the red and three dark lines in the green blue. In the position of H_β there was a pronounced darkening, resembling a sudden change of colour rather than a dark line. Possibly if definition had been better this might have appeared as a line or band. The violet end of the spectrum seemed unusually strong.

At 12.45 G.M.T. there was a dark line in the position of H_β , and it was suspected that there was a bright line on its red side. At 13.58 G.M.T. I considered that there were two bright lines in this region (probably H_β and 502), and that H_β had a dark border on the violet side.

June 9.—The Nova was the brightest star in the northern sky, and at least half a magnitude brighter than Vega.

There was no great change in the spectrum from the previous night, but there were undoubtedly indications of several bright, and some dark lines, which were all difficult to locate. Two dark lines were suspected at the extreme red end of the spectrum, and several dark lines in the green blue, with possibly bright edges towards the red. It was considered that H_β was plainly visible as a dark line at 11.23 G.M.T., when observations ceased.

June 10.—At 9.40 G.M.T. the Nova was equal in magnitude to Arcturus.

It was immediately apparent that intensely brilliant lines had developed in the spectrum, and that the majority of these lines had dark borders on the violet side. At 11.23 G.M.T. the principal lines seen were:—

H_α bright.

A faint bright band in red between H_α and D.

A brilliant line at about 517.

A brilliant line at about 502.

H_β very bright, with a dark border on violet side.

Two dark lines suspected between H_β and H_γ .

H_γ bright.

H_δ (?). A faint bright line suspected beyond H_γ .

Later in the night it was seen that there was a faint bright line at about 532, and a faint bright line was also seen in about the position of D, with a dark line on its violet side. As the

night advanced the intensity of the bright lines, especially of H_{α} , increased, and I am confident that I watched this increase taking place.

At 12.0 G.M.T. the Nova was estimated to be 0.5 mag. brighter than Vega, and 0.3 brighter than Arcturus.

At 12.10 G.M.T. the H_{α} line had become equal in brightness to H_{β} , which had previously been the brightest line in the spectrum, but half an hour later H_{β} was again the brighter of the two.

At 12.25 G.M.T. it was noted that there was a strong dark shading on the violet side of the bright line at 517, but later in the night I noted that this shading was not very pronounced. A faint bright line or band was seen at about 492.

At 13 hrs. G.M.T. the Nova was estimated to be equal to Arcturus.

June 11.—The Nova had faded since last night, and was now less than Vega, but still brighter than Altair.

The darkest line in connection with the group H_{β} , 502, 517, was that on the violet side of 502, and I am of the opinion that there was a change in this respect from the previous night. The violet end of the spectrum seemed to have faded relatively since June 8. The spectrum was a mass of dark and bright lines, the principal of which were:—

H_{α} very bright, with a broad moderately dark shading rather than line on its violet side.

A double bright band or line in the red orange, with a very faint dark line in its centre, and a dark border on its violet side.

A bright line in the position of D, with a dark border on the violet side.

In the pale green a faint hazy dark line, followed by a faint bright line.

A bright line at about 532.

517, 502, and H_{β} very bright.

A faint thin bright line about 492.

A dark line about a quarter of distance from H_{γ} to H_{β} .

H_{γ} bright.

Other lines, both bright and dark, were suspected, but could not be accurately located.

At 11.47 G.M.T. it was noted that there was then no very strong dark line on violet side of 517.

At 11.50 G.M.T. the Nova was estimated to be 0.25 mag. fainter than Arcturus. The colour was white.

Since the above date further developments and changes have taken place in the spectrum, but these will doubtless be recorded by photography.

I am indebted to Professor Fowler for the approximate wave lengths of the lines shown on my sketches and quoted above.

*Observations of Nova Aquilæ (1918) made at the Solar Physics
Observatory, Cambridge.*

(Communicated by the Director.)

A telephone message from the Astronomer Royal on Sunday morning, 1918 June 9, gave information of the discovery on the previous night, by Mr. W. H. Steavenson and others, of a Nova near α Aquilæ. Preparation was made for securing photographs of the spectrum with the new spectroscope attached to the Huggins telescope. Thanks to the careful work done and recorded by Captain Rolston before he went away to military duties, the adjustment was greatly facilitated. The instrument has since his departure been fitted with a convenient sparking apparatus for comparison spectra, and photographs of the spectrum of the Nova have been secured with comparison spectra on the three nights, June 9, 12, and 13.

1918 June 9, Sunday, 9.30–10.0 G.M.T.

The magnitude of the Nova was visually estimated at about one-fifth magnitude brighter than α Lyræ, and nearly a full magnitude brighter than α Aquilæ, no account being taken of the difference due to absorption at different altitudes. The Nova to the naked eye appeared remarkably blue-white, in spite of its low altitude.

The spectrum was observed visually through a direct vision prism and a 4-inch telescope. No signs of discontinuities could be detected in the brilliant continuous spectrum. The star, when observed with a direct vision prism without the use of the telescope, exhibited a spectrum in which, in comparison with α Lyræ, the red part was brighter relatively to the blue.

A photograph of the spectrum was attempted under very unfavourable circumstances; clouds frequently interrupted the exposure. The plate shows only a faint continuous spectrum extending over the region $\lambda\lambda$ 4000–4900; and the only discontinuity in this region is a doubtful absorption line between 4308 and 4325; no indication of any emission line has been detected in a hasty examination. The exposure was made between 10^h 12^m and 11^h 10^m G.M.T.

June 10, Monday, 9.30–10.30 G.M.T. The Nova was distinctly brighter than α Aquilæ, and the Nova at its altitude was estimated as equal to α Lyræ at its altitude.

The spectrum, visually studied with a direct vision prism and the 4-inch telescope, showed a not very conspicuous emission line near the position of H_{α} and a broadish absorption line near the position of H_{β} .

No photograph of the spectrum was attempted.

June 12, Wednesday. The conditions were less favourable. No estimate of magnitude was recorded.

Visual observations of the spectrum showed that the characteristic features of pairs of bright and dark lines were strongly marked, the H_{α} , H_{β} , and H_{γ} lines of hydrogen being easily seen.

Two photographs of the spectrum were secured between 9^h 19^m and 11^h 10^m G.M.T. The bright lines are seen to be conspicuous features relatively to the continuous spectrum; but the photographs are faint.

June 13, Thursday. A brilliant night from 9^h to 15^h G.M.T. The Milky Way was conspicuous, and the Nova was seen to be just on the following border of the dark rift which lies in the middle of the bifurcation of the Milky Way near Altair.

The brightness of the Nova had decreased considerably. It was estimated as about three-tenths of a magnitude less bright than Altair.

Visual observations of the spectrum showed a very complicated system of bright and dark lines. The brilliance of H_{α} was an outstanding feature, and it was interesting to note that when the Nova was viewed with direct vision prism without telescope the H_{α} line was bright enough to be recognised. Thus it must be admitted that this one component of the total light of the star is probably brighter than a star of the fifth magnitude.

Three photographs of the spectrum were secured, all of them with comparison spectrum from an iron spark; and one of them has a spectrum of α Cygni imposed alongside of the spectrum of the Nova. The juxtaposition of these two spectra serves to point the resemblances and differences in the various stellar lines; and it will be seen that many of the features of the spectrum of α Cygni are reproduced in the spectrum of the Nova.

Solar Physics Observatory :
1918 June 14.

The Stellar Magnitude Scales of the Astrographic Catalogue.
Twelfth Note: The Hyderabad Magnitudes (-17° and -18°), the Perth Magnitudes (-32° and -34°), the Edinburgh Magnitudes (-38°), and the Cape Magnitudes (-41° and -42°). By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The material here presented is in part new, in part old material revised on the plan introduced in the eleventh note. Before that note was published the Hyderabad zone -17° and the Perth zone -32° represented all the available information between the Equator and -41° ; we have now, thanks to the kind-

ness of various directors of observatories, -15° from Tacubaya, -18° from Hyderabad, -25° from Cordoba, -34° from Perth, and -38° from Edinburgh, which with the Cape zones give a good preliminary survey of the greater part of the southern hemisphere. It was at first hoped to attempt something of a definite collation of the whole material, but it seems desirable to defer this still further until some additional material for the southern hemisphere has been presented and the northern hemisphere revised.

2. Meanwhile the following remark may be made. In the first ten notes attention was mainly directed to the *differences* between the columns of the tables: representing ratios of faint stars to bright. The excellent approximation to the galactic condensation deduced from a portion of the material by Mr. Seares makes it possible to clear each column separately from the greater part of this condensation, according to the plan described in the eleventh note, which will be adopted in what follows. But it must not be overlooked that these separate columns are affected by accidental errors, from which the differences formerly studied are largely free. An example from what follows will make this clear.

3. Below (see Tables X. and XI.) are given the harmonics for the Cape zones -41° and -42° ; two zones which overlap to the extent of half their own width, and obviously have in all respects much in common. Compare, for brevity, the coefficients of $\sin \theta$ in the first, and of $\cos 2\theta$ in the second harmonics:—

Cape -41° .			Cape -42° .		
log N_0 .	$\sin \theta$.	$\cos 2\theta$.	log N_0 .	$\sin \theta$.	$\cos 2\theta$.
0.74	+ .148	- .016	9.97	+ .080	- .108
1.04	+ .132	- .003	0.68	+ .068	- .143
1.50	+ .058	- .014	1.70	+ .039	- .104
1.84	+ .026	- .041	1.98	+ .013	- .151
2.37	- .024	- .016	2.41	- .083	- .156
Mean	+ .068	- .018		+ .023	- .132

The differences in log N_0 are due simply to the methods of counting, and are not otherwise significant. But there is a striking change in the *mean* value of the coefficient of $\cos 2\theta$, from $- .018$ to $- .132$. The separate values are consistent in showing little change with log N_0 in both zones, but the two zones differ remarkably in the mean value. The coefficient of $\sin \theta$ shows a steady drop in both cases; and though there are accidental deviations, the differences for the available corresponding extremes

$$\text{Zone } -41^\circ, 0.74 \text{ to } 2.37 = +.148 + .024 = +.172$$

$$\text{Zone } -42^\circ, 0.68 \text{ to } 2.41 = +.080 + .083 = +.163$$

are fairly consistent. But the mean values for the two zones show a sensible difference.

4. Now, scrutiny of the figures shows that these differences are due to local differences in the counts, which may be ascribed to a batch of specially sensitive plates for one zone, or to a spell of exceptional weather. The most striking instance is for the portion $17^h - 18^h$.

Comparison of $17^h - 18^h$ in Zones -41° and -42° .

Zone.	Total Stars.	log.	Dates.
-41°	11,886	4.07	9 plates in 1898 June-Aug., 3 in 1901
-42°	22,055	4.34	All plates in 1901 July-Sept.

No information is to hand throwing any light on this difference, which is to be regarded as "accidental" in the sense that the sensitivity of the plates used, or the weather conditions, or both combined, varied in at least the ratio 2 to 1 over long periods. Hence the results for any given magnitude will be affected by the whole extent of these variations. But the *differences* between one magnitude and another may yet be largely independent of these accidental variations, and such figures as those above quoted encourage us to hope that they are.

5. We conclude that though there is a convenience in presenting the results for the columns (corresponding to selected magnitudes) separately, we should still direct attention chiefly to the *differences* between them, as being free from errors which affect the separate columns themselves. The same applies, of course, to the harmonics calculated from the columns. I proceed to give the new material.

The Hyderabad Zones for -17° and -18° .

6. The original counts for zones -17° and -18° are given in *M.N.*, 77, 434, and 78, 368. They were reduced to standard shape

- (1) by the formation of $\log N_\gamma$ for one square degree;
- (2) by the deduction of $\log N_0$ from

$$\log N_0 = \log N_\gamma (1 + .0025\gamma) K_\gamma,$$

where γ is the galactic latitude in degrees, and N_0 is the corresponding value of N_γ on the galactic equator;

- (3) by the formation of the mean value of $\log N_0$, which is used as an index as it stands, no attempt being made to convert it into stellar magnitude at present;
- (4) by the formation of residuals from this mean value.

This process is fully illustrated in *M.N.*, 78, 54, a table of K_γ being given (p. 56); and there is, consequently, no need to give here anything beyond the mean value (3) and the residuals specified in (4). If the original counts are required for these or other zones given subsequently, they can be recovered by inverting the above steps.

7. It will be seen that we have five closely corresponding sets for zones -15° , -17° , and -18° as follows:—

TABLE I.

Mean Values of $\log N_0$.

Zone.	α .	β .	γ .	δ .	ϵ .
-15°	0.62	1.22	1.76	2.16	2.27
-17	0.68	1.46	1.79	2.08	2.31
-18	0.70	1.51	1.81	2.08	2.32
Mean	0.67	1.40	1.79	2.11	2.30

TABLE II.

Observed Deviations of $\log N_0$ in units of 0.01 for Hyderabad Zones -17° and -18° .

R.A.	Zone -17° .						Zone -18° .					
	Values of d (Hyd.).					Gal. Lat.	Values of d (Hyd.).					Gal. Lat.
	50.	30.	20.	12.	8.		50.	30.	20.	12.	8.	
0	+19	+35	+28	+20	+15	-79°	+12	+17	+20	+21	+12	-80°
1	-3	+14	+8	+6	+4	-75	-8	+11	+14	+18	+9	-75
2	-11	+9	+11	+12	+4	-63	+20	+19	+23	+18	+12	-63
3	-7	-5	+4	+2	-8	-50	+32	+18	+15	+11	+6	-50
4	+6	-8	0	-2	-4	-37	+22	+21	+19	+13	+8	-37
5	+37	+29	+18	+7	-8	-24	-18	-5	-10	-7	-8	-23
6	+16	+3	+2	-5	-14	-11	-8	-15	-16	-16	-16	-10
7	-7	-1	+11	+13	+13	+2	-18	-20	-12	-5	+1	+2
8	-9	-7	-5	-4	-3	+14	-37	-41	-33	-28	-15	+14
9	-14	-21	-15	-16	-14	+25	0	-18	-17	-6	+3	+25
10	-23	-25	-22	-17	-14	+35	+27	+13	+9	+8	+14	+34
11	-18	-20	-13	-6	+3	+42	+11	+8	+3	-3	-1	+41
12	-3	-7	-9	-3	-4	+45	+9	+8	+5	+2	+12	+44
13	+15	+7	+4	+6	+13	+44	+10	+7	-3	-6	+6	+42
14	+14	+20	+17	+14	+23	+38	+9	+3	0	-6	0	+37
15	+3	-1	+4	+5	+13	+29	+1	+16	+4	-5	-6	+29
16	-5	-19	-29	-31	-34	+18	-21	-24	-31	-38	-39	+18
17	+1	-15	-21	-20	-8	+7	-15	-22	-19	-22	-3	+7
18	-23	-26	-28	-26	-8	-5	+7	+3	+7	+7	+11	-6
19	-17	-37	-18	-13	-6	-18	-30	-40	-25	-15	-19	-18
20	-2	+9	+10	+10	+14	-31	-11	+2	+5	+4	-13	-31
21	+19	+32	+22	+16	+12	-44	+5	+22	+25	+21	+10	-45
22	+14	+15	+14	+11	+11	-57	+14	+16	+18	+18	+7	-58
23	0	+25	+17	+13	+10	-71	-2	+7	+11	+7	-1	-71
Mean $\log N_0$	0.68	1.46	1.79	2.08	2.31		0.70	1.51	1.81	2.08	2.32	

8. Now it is pretty clear that there are systematic features in these residuals themselves, as distinguished from their differences. The portion $20^h - 4^h$ is generally positive, when that for $5^h - 19^h$ is negative; and this feature appears also in the results for -15° (Tacubaya) given in the eleventh note. It is quite possible that this feature, common to results obtained in two distant observatories, is celestial, and not terrestrial in origin. But it is subject to the disability just noticed, and for the moment is not under special attention. We are concerned chiefly with the differences between the columns, and it will be seen that there is actually (either by chance or association) a broad distinction between those of the two portions just named. [The lines of division for the differences are, however, not quite the same, and the evidence of other zones suggests that the near correspondence in the present case is due to chance.]

9. If we divide the circuit of R.A. into two portions, $21^h - 5^h$ and $6^h - 20^h$, and take the means of the residuals for these separately, we get the following results, to which are added those of -15° for comparison. The sets are designated by the letters shown in Table I.

TABLE III.

Division of the Circuit into two Portions according to Differences of Columns.

Zone.	Portion $21^h - 5^h$ R.A.				
	$\alpha.$	$\beta.$	$\gamma.$	$\delta.$	$\epsilon.$
-15°	+ 14	+ 12	+ 8	+ 1	+ 1
- 17	+ 8	+ 16	+ 12	+ 9	+ 4
- 18	+ 9	+ 14	+ 15	+ 13	+ 6
Mean	+ 10	+ 14	+ 12	+ 8	+ 4
	Portion $6^h - 20^h$ R.A.				
	$\alpha.$	$\beta.$	$\gamma.$	$\delta.$	$\epsilon.$
- 15	- 12	- 10	- 7	- 1	0
- 17	- 5	- 9	- 7	- 6	- 2
- 18	- 4	- 8	- 8	- 9	- 6
Mean	- 7	- 9	- 7	- 5	- 3
Difference	+ 17	+ 23	+ 19	+ 13	+ 7

10. The mean residuals in one portion are positive and in the other negative, but, as already remarked, this is not the feature under consideration. We see that in $21^h - 5^h$ the residuals diminish from β to ϵ , while in $6^h - 20^h$ they increase. The group α seems to show a reversal, but this is not confirmed by zone -15° , and may be an accident due to the paucity of bright stars. Omitting α for the present, we should say that the portion

$21^h - 5^h$ is "obscured" (fewer faint stars) relatively to $6^h - 20^h$, and the middle of the obscured patch would be about $1^h.5$ or 22° . Harmonic analysis of the residuals is as below.

TABLE IV.

*Harmonic Analysis of the Residuals of Table II.**Zone -17° .*

<i>d.</i>	$\log N_0$	$\sin \theta$	$\cos \theta$	$\sin 2\theta$	$\cos 2\theta$	$\sin 3\theta$	$\cos 3\theta$	$\sin 4\theta$	$\cos 4\theta$
50	0.68	+0.18	+0.52	+0.39	-0.08	-0.157	+0.035	-0.010	+0.035
30	1.46	+0.39	+0.131	+0.38	+0.85	-0.142	+0.083	+0.033	+0.028
20	1.79	+0.57	+0.113	+0.16	+0.68	-0.120	+0.043	+0.043	-0.011
12	2.08	+0.42	+0.83	+0.09	+0.78	-0.092	+0.023	+0.055	-0.022
All	2.31	-0.30	+0.42	-0.03	+0.74	-0.061	+0.016	+0.073	-0.022
Diff.	1.14	-0.84	-0.119	-0.48	-0.001	+0.109	-0.087	+0.052	-0.061
	1.00	-0.74	-0.104	-0.42	-0.001	+0.096	-0.076	+0.046	-0.053

Zone -18° .

50	0.70	+0.27	+0.17	+0.83	+0.112	+0.014	-0.082	-0.104	-0.010
30	1.51	+0.07	+0.82	+0.99	+0.125	-0.018	-0.052	-0.089	-0.002
20	1.81	+0.10	+0.121	+0.52	+0.118	+0.003	-0.052	-0.053	-0.004
12	2.08	+0.32	+0.122	+0.06	+0.108	+0.016	-0.050	-0.031	-0.010
All	2.32	+0.36	+0.028	+0.06	+0.89	+0.39	-0.034	-0.016	+0.012
Diff.	1.08	+0.51	-0.053	-0.139	-0.046	+0.070	+0.020	+0.095	+0.008
	1.00	+0.47	-0.049	-0.129	-0.043	+0.065	+0.019	+0.088	+0.007

11. To obtain the rate of change the top row has been excluded as before, and the sum of second and third rows subtracted from sum of fourth and fifth. It is by no means intended to reject the evidence of the brighter stars finally; but we shall simplify the issue for the present in confining attention to the more numerous stars (from about $\log N_0 = 1.20$ to 2.20). The row "Diff." shows the result of the subtraction, and in the row below the numbers are reduced to unit of $\log N_0$. In the eleventh note the first line was included for Tacubaya -15° ; but it is easy to revise the calculation so as to accord with that above, and the three results then are—

	$\sin \theta$	$\cos \theta$	R.	θ
-15°	+0.17	-0.92	-0.093	350°
-17	-0.74	-0.104	-0.128	35
-18	+0.47	-0.49	-0.068	316
Mean	-0.003	-0.082	-0.082	2

The Perth results for -32° and -34° .

12. The Perth counts for -32° were given in the fifth note (*M.N.*, 75, 144), and the magnitude scale was there shown to approach uniformity very closely, except that classes K, L, M should be combined into one class. Special interest, therefore, attaches to the counts for -34° (a completely independent zone), which were kindly supplied by Mr. Curlewis some months ago; and even in these days of restricted printing they may be given in full (Table V.), except that classes K, L, M have been added together as class M.

13. In Table VI. are given the residuals from a mean $\log N_0$ deduced by the process fully illustrated in the eleventh note.

TABLE V.

*Perth Star Counts for 10 Plates in Zone -34° .**(For -32° see M.N., 75, 144.)*

R.A.	Brt.	A.	B.	C.	D.	E.	F.	G.	H.	J.	M.	Total.
0	3	4	6	13	27	64	69	107	163	163	182	801
1	1	1	13	17	43	74	116	171	157	170	167	930
2	7	7	20	28	40	80	115	220	250	243	205	1215
3	7	6	11	26	66	117	173	221	263	259	266	1415
4	20	13	23	44	87	209	314	383	471	380	472	2416
5	23	30	32	53	90	190	278	442	608	537	448	2731
6	37	43	37	94	153	287	384	566	767	811	907	4086
7	57	60	63	151	257	553	771	1179	1724	1494	2154	8463
8	35	33	57	98	285	542	820	1703	1952	2188	2830	10543
9	27	23	21	79	110	279	411	693	1001	809	1049	4502
10	16	20	17	70	117	220	356	546	743	671	883	3659
11	20	13	13	38	80	148	193	452	460	532	802	2751
12	20	21	14	23	70	147	196	580	469	593	803	2936
13	21	30	26	41	107	240	260	594	659	681	1139	3798
14	20	14	16	33	83	227	301	607	710	939	1442	4392
15	19	20	17	36	69	180	176	541	509	627	896	3090
16	37	46	19	59	144	321	374	824	1240	894	1559	5517
17	112	77	93	130	348	637	758	1782	1617	2373	4030	11957
18	63	59	74	104	160	551	630	1297	1809	1933	2587	9267
19	23	23	27	67	96	264	333	537	857	784	1033	4044
20	15	17	20	42	80	193	297	377	605	612	894	3152
21	7	7	21	21	50	147	184	350	393	374	584	2138
22	20	6	14	23	50	120	151	284	323	284	410	1685
23	7	12	8	32	40	93	140	238	307	323	310	1510
Total	617	585	662	1322	2652	5883	7800	14694	18057	18674	26052	96998

14. The points Brt., B, D, F, H, M were selected for analysis, and the following are the residuals:—

TABLE VI.

Residuals of $\log N_0$ for Perth Zone -34° , in units of 0.01.

R.A.	Brt.	B.	D.	F.	H.	M.	Gal. Lat.
0	- 74	- 56	- 38	- 30	- 28	- 26	- 83°
1	- 130	- 47	- 20	- 13	- 16	- 19	- 78
2	- 26	- 5	- 5	- 10	- 7	- 11	- 66
3	- 26	- 24	- 3	+ 1	- 5	- 10	- 53
4	+ 21	+ 13	+ 14	+ 20	+ 12	+ 6	- 41
5	+ 18	+ 20	+ 10	+ 4	+ 2	- 4	- 29
6	+ 26	+ 20	+ 15	+ 4	- 4	- 6	- 17
7	+ 35	+ 28	+ 24	+ 18	+ 13	+ 12	- 6
8	+ 12	+ 11	+ 16	+ 15	+ 17	+ 18	+ 4
9	+ 6	- 8	- 6	- 5	- 6	- 9	+ 13
10	- 8	- 11	+ 2	0	- 2	- 4	+ 21
11	+ 8	- 12	- 9	- 15	- 12	- 8	+ 26
12	+ 10	- 1	- 11	- 14	- 5	- 2	+ 28
13	+ 11	+ 14	+ 7	+ 4	+ 6	+ 8	+ 27
14	+ 5	- 10	- 11	- 4	0	+ 9	+ 23
15	- 4	- 13	- 22	- 27	- 21	- 18	+ 17
16	+ 17	+ 4	- 4	- 9	- 4	- 6	+ 8
17	+ 62	+ 44	+ 32	+ 19	+ 15	+ 22	- 2
18	+ 44	+ 36	+ 18	+ 17	+ 20	+ 23	- 12
19	+ 13	+ 8	+ 6	+ 6	+ 6	+ 7	- 24
20	+ 5	+ 6	+ 6	+ 12	+ 10	+ 14	- 36
21	- 27	- 8	- 9	+ 1	+ 5	+ 6	- 48
22	+ 26	+ 3	- 1	+ 2	+ 5	+ 5	- 61
23	- 28	- 17	- 6	- 2	+ 3	+ 4	- 74
Mean $\log N_0$ }	9.83	0.41	0.97	1.55	2.00	2.27	

15. We can now make Table VII. on the same plan as Table IV., adding details for Perth zone -32° , deduced from the numbers given in the fifth note (*M.N.*, 75, 144):—

TABLE VII.

Harmonic Analysis of Residuals for -32° and -34° .

<i>Zone -32°.</i>									
Class.	$\log N_0$.	$\sin \theta$.	$\cos \theta$.	$\sin 2\theta$.	$\cos 2\theta$.	$\sin 3\theta$.	$\cos 3\theta$.	$\sin 4\theta$.	$\cos 4\theta$.
Brt.	9.89	-.083	-.168	+.070	-.021	+.040	+.121	+.062	+.103
B	0.58	-.047	-.072	-.001	-.008	+.108	+.038	+.038	+.048
D	1.07	-.046	-.050	-.019	+.015	+.083	+.045	+.039	+.063
F	1.57	-.031	-.032	.000	+.021	+.083	+.045	+.063	+.061
H	1.98	-.057	-.038	-.007	+.012	+.080	+.037	+.060	+.041
M	2.25	-.088	-.043	-.024	-.016	+.068	+.032	+.062	+.032
Diff.	1.59	-.068	+.001	-.012	-.040	-.018	-.021	+.020	-.051
	1.00	-.043	+.000	-.008	-.025	-.011	-.013	+.013	-.032
<i>Zone -34°.</i>									
Brt.	9.83	-.115	-.243	-.140	-.285	-.092	-.061	-.058	+.085
B	0.41	-.046	-.092	-.070	-.215	-.025	-.028	-.035	+.045
D	0.97	+.020	-.035	-.059	-.150	+.008	-.028	+.038	+.012
F	1.55	+.016	+.004	-.063	-.113	-.002	-.038	+.032	-.018
H	2.00	-.018	-.008	-.060	-.081	-.008	-.022	+.032	-.012
M	2.27	-.054	-.022	-.065	-.072	-.013	-.019	+.048	-.001
Diff.	1.75	-.108	+.001	-.003	+.110	-.027	+.025	+.010	-.007
	1.00	-.061	+.001	-.002	+.063	-.015	+.014	+.057	-.004

16. These figures show clearly—

(a) That the *mean values* of the coefficients for all values of $\log N_0$ are affected by considerable accidental errors, as remarked in §§ 2-5. For the third harmonic, for instance, the coefficients of both $\sin 3\theta$ and $\cos 3\theta$ are consistently positive for zone -32° , consistently negative for zone -34° . We conclude that since two zones so closely contiguous differ essentially, the mean values of both are without significance, and must be attributed to accidental causes.

(b) But the *differences* between the rows are more significant, because the accidental causes are largely eliminated. In the row marked "Diff." the sum of D + F is subtracted from the sum of H + M. The exclusion of the brighter stars is not meant to be final; they are put aside, partly because the results are affected by larger accidental errors, and partly because attention has hitherto been directed chiefly to the fainter stars represented by rows D to M. It has already been remarked in § 10 that the brighter stars may behave differently, and this behaviour must ultimately receive careful attention. But for the present we simplify the problem by putting them aside.

(c) In the bottom row the differences $M + H - F - D$ are divided by the factors 1.59 and 1.75 respectively so as to form the gradient per unit of $\log N_0$, and we may fairly say that the first harmonic is more consistent and more pronounced than the

others. It is undoubtedly small in this region of the sky, but is still clearly traceable. In the notation of § 11 we have

For zone -32°	$R = -.043$	$\theta_0 = 90^\circ$
„ -34	$R = -.061$	$\theta_0 = 91$
Mean of both	$R = -.052$	$\theta_0 = 90^\circ$

The Edinburgh Zone -38° .

17. The plates measured at Edinburgh were taken at Perth, and form part of the Perth zones. But the stellar magnitudes have been inferred from measurements of diameter, not by comparison of standard images, as at Perth. Professor Sampson has expressed his preference for the Perth method, and proposes to revise his measures in accordance with it (*M.N.*, 77, 618). He therefore does not give the original counts, but he has kindly supplied me with his counts for zone -38° , which may be printed on the understanding above indicated, that Professor Sampson is not himself fully satisfied with them, and that they are subject to revision. The plates in each row have centres at 0^m , 10^m , . . . 50^m , and the counts are cumulative, being down to and including the diameters specified.

TABLE VIII.

Edinburgh Star Counts for 6 Plates in Zone -38° .

R.A.	$d=50.$	40.	30.	20.	10.	All.
0	8	37	58	203	665	1050
1	19	46	84	249	873	1271
2	10	35	69	181	643	994
3	12°	28	65	189	615	996
4	17	38	81	278	994	1403
5	12	40	81	326	1088	1638
6	29	78	129	413	1734	2592
7	70	144	220	622	2322	3550
8	50	144	244	831	3231	5319
9	35	139	430	808	2630	4126
10	27	89	238	600	2168	3670
11	11	58	102	298	1169	2112
12	17	47	87	271	1049	1886
13	16	40	82	337	1638	3138
14	17	32	71	345	1947	4047
15	20	44	70	340	2024	4273
16	30	63	122	365	1393	2429
17	37	99	227	908	4549	8490
18	18	70	121	486	3079	6238
19	25	43	86	299	1566	2817
20	16	37	70	353	1531	2456
21	22	47	85	251	825	1412
22	10	27	58	208	851	1321
23	8	37	91	249	773	1225
Total	536	1462	2971	9410	39357	68453

18. In view of Professor Sampson's expressed opinion about these numbers, it seems undesirable to give the detailed residuals here; but they were formed and analysed in the same way as the others, and we proceed to give the deduced harmonics.

TABLE IX.

Harmonic Analysis of Residuals for Zone -38° .

<i>d.</i>	$\log N_0$	$\sin \theta$	$\cos \theta$	$\sin 2\theta$	$\cos 2\theta$	$\sin 3\theta$	$\cos 3\theta$	$\sin 4\theta$	$\cos 4\theta$
50	0.11	+0.57	-0.02	-0.89	-0.85	+0.26	+0.08	+0.33	-0.15
40	0.60	+0.99	+0.33	-0.103	+0.16	+0.83	+0.37	+0.12	+0.13
30	0.92	+0.111	+0.43	-0.106	+0.48	+0.107	+0.13	-0.025	-0.009
20	1.48	+0.41	+0.76	-0.065	+0.18	+0.44	0.00	+0.23	0.00
10	2.13	-0.32	+0.73	-0.42	-0.009	+0.37	-0.10	+0.53	+0.04
All	2.37	-0.83	+0.52	-0.28	-0.003	+0.35	-0.09	+0.57	+0.08
Diff.	2.10	-0.267	+0.06	+0.101	-0.78	-0.79	-0.32	+0.112	+0.21
	1.00	-0.127	+0.03	+0.48	-0.37	-0.38	-0.15	+0.53	+0.10

The explanations are the same as for Table VII., the "Diff." being taken between rows (6) + (5) - (4) - (3).

The first harmonic gives $R = -0.127$ $\theta_0 = 91^\circ$.

The Cape results for -41° and -42° .

19. The detailed counts for -41° are given in *M.N.*, 75, 59. The harmonics deduced from the residuals, cleared from galactic condensation as before, are as below:—

TABLE X.

Harmonic Analysis of Residuals for Zone -41° .

<i>d.</i>	$\log N_0$	$\sin \theta$	$\cos \theta$	$\sin 2\theta$	$\cos 2\theta$	$\sin 3\theta$	$\cos 3\theta$	$\sin 4\theta$	$\cos 4\theta$
200	0.74	+0.148	+0.06	-0.58	-0.16	-0.24	+0.66	-0.04	+0.31
150	1.04	+0.132	+0.41	-0.57	-0.03	+0.13	+0.51	+0.21	+0.52
110	1.50	+0.58	+0.03	-0.49	-0.14	+0.23	+0.51	+0.36	+0.29
80	1.84	+0.26	+0.04	-0.81	-0.41	+0.02	+0.26	+0.36	+0.58
All	2.37	-0.24	-0.22	-0.32	-0.16	-0.01	+0.21	+0.06	+0.27
Diff.	2.43	-0.278	-0.65	+0.02	-0.38	+0.12	-0.70	+0.25	+0.02
	1.00	-0.115	-0.27	+0.01	-0.16	+0.05	-0.28	+0.10	+0.01

For the first harmonic $R = 0.118$ $\theta_0 = 77^\circ$.

The comparison is here made between the first two rows and the last two.

20. For zone -42° some figures for the fainter stars are given in *M.N.*, 76, 3. But the march in $\sin \theta$ for -41° is so large and

consistent that it seemed desirable to extend the counts for -42° to brighter stars. It is tolerably easy to count all those measured at the Cape with zero for integer; with unity for integer; and so on. These counts were made for zone -42° , and are being made for zone -43° . [For other zones we cannot yet go beyond the fainter stars, for which counts were kindly supplied in MS. by Mr. Hough, since no other volumes are yet printed.]

TABLE XI.

Harmonic Analysis of Residuals for Zone -42° .

<i>d.</i>	$\log N_0$	$\sin \theta$	$\cos \theta$	$\sin 2\theta$	$\cos 2\theta$	$\sin 3\theta$	$\cos 3\theta$	$\sin 4\theta$	$\cos 4\theta$
3'00	9'97	+ '080	- '018	- '021	- '108	- '057	+ '014	+ '090	- '010
2'00	0'68	+ '068	+ '013	- '038	- '143	- '010	- '002	+ '045	+ '023
1'00	1'70	+ '039	+ '021	- '069	- '104	+ '031	- '015	+ '016	- '018
[0'00]	1'98	+ '013	+ '062	- '057	- '151	+ '048	- '006	- '001	+ '001
All	2'41	- '083	+ '025	- '048	- '156	+ '046	- '023	+ '017	+ '029
Diff.	2'01	- '177	+ '053	+ '002	- '060	+ '073	- '012	- '045	+ '025
	1'00	- '088	+ '026	+ '001	- '030	+ '036	- '006	- '022	+ '012

For first harmonic $R_0 = .092$ $\theta_0 = 106^\circ$.

21. Collecting the three zones -38° , -41° , and -42° we have

	$\sin \theta$	$\cos \theta$	<i>R.</i>	θ_0
-38	- '127	+ '003	- '127	91°
-41	- '115	- '027	- '118	77
-42	- '088	+ '026	- '092	106
Mean	- '110	+ '001	- '110	91

The revision of the other zones of the southern hemisphere may be deferred until Cape -43° has been fully counted. The results so far may be thus summarised:—

Dec.	No. of Zones.	$\sin \theta$	$\cos \theta$	<i>R.</i>	θ_0	Spiral.	Diff.
-17°	3	- '003	- '082	- '082	2°	309°	+ 53°
-25	1	- '024	+ '070	- '074	161	339	+ 182
-33	2	- '052	+ '000	- '053	90	8	+ 82
-40	3	- '110	+ '001	- '110	91	34	+ 57

The column headed "Spiral" has been deduced from the formula

$$\theta_0 = 247^\circ - 3.66\delta.$$

22. The single zone -25° is discordant; and the other three determinations show that the suggested spiral must be modified, in this part of the heavens at any rate. It is natural to inquire how

far this modification results from the removal of the galactic condensation. This condensation consists of two parts:—

(a) A constant part K_γ , applicable to all values of $\log N_0$ alike, tabulated on p. 57 of this volume. This does not affect the differences here being studied.

(b) The correction

$$+ \cdot 0025\gamma \log N_\gamma$$

taken as simply proportional to the galactic latitude γ , a simple supposition which was outlined in 1915 June, and which apparently fits very closely the figures recently obtained by Mr. Seares.

It is easy to tabulate the effect of this latter term for a change of unity in $\log N_\gamma$, *i.e.* to tabulate the first harmonic of

$$+ \cdot 0025\gamma.$$

Representing it in the form

$$A \sin \theta + B \cos \theta,$$

then the ratio A/B is constant and equal to $\tan \theta^h 40^m = 0.18$, if the R.A. of the galactic pole be $12^h 40^m$.

23. The coefficients obtained by a process strictly comparable with that used for $\log N_0$ are:—

$\delta =$	$+10^\circ$	$+20^\circ$	$+30^\circ$	$+40^\circ$	$+50^\circ$	$+60^\circ$	$+70^\circ$	$+80^\circ$
B	$-\cdot 024$	$-\cdot 044$	$-\cdot 061$	$-\cdot 069$	$-\cdot 073$	$-\cdot 071$	$-\cdot 050$	$-\cdot 026$
A	$-\cdot 004$	$-\cdot 008$	$-\cdot 011$	$-\cdot 012$	$-\cdot 013$	$-\cdot 013$	$-\cdot 009$	$-\cdot 005$

with reversed sign for southern declinations.

24. Hence if we had not first cleared the results for galactic condensation, they would have stood as below (approximately):—

Zone.	$\sin \theta.$	$\cos \theta.$	$\theta_0.$	Spiral.	Diff.	Effect of Gal.
-17°	$-\cdot 009$	$+\cdot 120$	5°	309°	$+ 56^\circ$	$- 3^\circ$
-25	$-\cdot 033$	$+\cdot 018$	118	339	$+ 139$	$+ 43$
-33	$-\cdot 063$	$-\cdot 064$	45	8	$+ 37$	$+ 45$
-40	$-\cdot 121$	$-\cdot 068$	61	34	$+ 27$	$+ 30$

It seems possible that if this material had been earlier available, some correction to the spiral would have been suggested independently of the galactic correction. But the revision may now be deferred until other zones have been corrected.

Thirteenth Note on the Number of Faint Stars with Large Proper Motions. Zones +26° and +27°. By F. A. Bellamy, M.A.

This note clears off all the plates that have been measured and that are suitable for comparison for proper motions. Reference to the former notes are given in the Twelfth Note, *Monthly Notices*, 78, 471.

As shown in Table I., two of the five plates now included have intervals between the two pairs of less than ten years. They are interesting regions. The other three pairs have twenty-three years' interval. The region about +26° 4^h 24^m is the barest region in the whole of the Oxford seven zones.

TABLE I.
Stars with Probable Large Proper Motions.

Zone. R.A. Plate Centre.	Total Stars Compared.	Interval in Years.	Total P.Ms.	Centennial P.Ms.	
				Above 15".	Below 15".
h m					
+26° 2 56	199	23'15	11	3	8
+26 4 16	116	23'15	3	0	3
+27 4 20	28	8'08	0	0	0
+26 7 4	243	22'98	5	2	3
+27 11 48	84	9'22	1	1	0
5 plates	670	...	20	6	14

From the comparison of these 670 stars, only 20 stars with proper motions above 6".5 in a century have been found; of these, 8 are stars in the Cambridge *A.G.C.*, but none visible to the unaided eye, and all are under half a second of arc in a year. The results are disappointing, though useful.

The same methods and limits in former notes in this series have been followed. One plate, +27° 11^h 48^m, was very accurately measured at H.H. the Nizam's Observatory at Hyderabad by D. R. Sripathi Rao in 1915.

In the comparison of plates 255 (1892.947) and 2932 (1916.101) the star in Oxford +26° 7246 was not found on the later and slightly better plate in point of number measured. In the examination of the plate 255 a stellar object is certainly still there—the plate was measured twenty-one years ago—but the separation between the two images in y is slightly less than it should be. May it be a planet?

Cambridge *A.G.C.* 3745 and 3746 (Oxford +26°, 20264, 20265). In our ledgers for Oxford-Cambridge, and in the *Oxford Astrographic Catalogue*, viii. pt. 1, p. 31, these two stars gave unusually large residuals in x , -'037 + '001 and -'022 + '003. The comparison of the two plates 300 (1893.121) and 2936 (1916.101) shows that the large residuals in x must be due to the Cambridge observations, since the Oxford Δx and Δy are

in twenty-three years, for 3745 $- \cdot 002 + \cdot 001$, and for 3746 $+ \cdot 001 + \cdot 002$ (unit = $0'' \cdot 3$).

In $+26^{\circ} 7^h 4^m$, Oxford 20286 should be omitted; it is the same as 20285.

TABLE II.
List of P.Ms.

Plate Centre.			Oxford.		A.G.C. No.	Δx . ($\cdot 001 = 0'' \cdot 3$)	Δy .	Cent. P.M. in Arc.	Interval in Years.
	h	m	No.	ϕ Mag.					
$+26^{\circ}$	7	4	20123*	8'9	3831	$- \cdot 032$	$- \cdot 004$	42'0	22'98
$+26$	2	56	7181	10'8	...	$+ \cdot 017$	$- \cdot 017$	31'1	23'15
$+27$	11	48	28359	11'7	...	$+ \cdot 008$	$\cdot 000$	26'1	9'22
$+26$	2	56	7233*	7'6	1553	$+ \cdot 017$	$- \cdot 010$	25'6	23'15
$+26$	2	56	7247*	9'2	1530	$- \cdot 001$	$+ \cdot 014$	18'2	23'15
$+26$	7	4	20192*	7'5	3774	$- \cdot 007$	$- \cdot 013$	14'8	22'98
$+26$	2	56	7201	11'7	...	$- \cdot 003$	$- \cdot 007$	9'9	23'15
$+26$	7	4	20176*	9'4	3814	$- \cdot 005$	$- \cdot 005$	9'2	22'98
$+26$	2	56	7179	10'1	...	$- \cdot 007$	$- \cdot 001$	9'2	23'15
$+26$	2	56	7172	12'1	...	$+ \cdot 005$	$- \cdot 004$	8'3	23'15
$+26$	4	16	10336	11'2	...	$+ \cdot 003$	$- \cdot 005$	7'6	23'15
$+26$	2	56	7116	10'3	...	$- \cdot 002$	$- \cdot 005$	7'0	23'15
$+26$	2	56	7318*	9'3	1534	$- \cdot 005$	$+ \cdot 002$	7'0	23'15
$+26$	7	4	20172	11'5	...	$+ \cdot 005$	$+ \cdot 001$	6'7	22'98
$+26$	7	4	20162*	10'9	3768	$- \cdot 003$	$- \cdot 004$	6'5	22'98
$+26$	2	56	7167	11'9	...	$+ \cdot 003$	$- \cdot 004$	6'5	23'15
$+26$	2	56	7175	11'7	...	$+ \cdot 005$	$\cdot 000$	6'5	23'15
$+26$	2	56	7218*	8'8	1522	$+ \cdot 004$	$+ \cdot 003$	6'5	23'15
$+26$	4	16	10380	11'3	...	$+ \cdot 003$	$- \cdot 004$	6'5	23'15
$+26$	4	16	10413	10'9	...	$+ \cdot 004$	$- \cdot 003$	6'5	23'15

University Observatory, Oxford:
1918 June 14.

On the Measurement of Time to the Thousandth of a Second.
By Professor R. A. Sampson, F.R.S., Astronomer Royal
for Scotland.

All time errors are relative quantities, for they arise from the comparison of two standards, whether these are merely two clocks, or a clock and a stellar transit, or otherwise. Hence when an irregularity appears the question arises which is technically known as "bringing home the guilt." When the matter is looked into more closely the problem becomes more intricate, because in the

best-established methods the actual determination from any time-keeper is highly indirect. To take for definiteness the method employed at Edinburgh for registering time from the standard mean time clock, Riefler No. 258. The pendulum is the ultimate standard; this releases the scape wheel; the scape wheel carries on the same arbor an auxiliary wheel which throws a loose pivoted arm into or out of contact with a stop, closing or opening an electric circuit; the current in this circuit being too feeble to operate the chronograph directly, operates instead a relay; the relay current goes through one of the electromagnets of the Fuess chronograph, throwing down a somewhat massive arm; this is furnished with a pricker which marks a paper tape; the tape is moved forward by clockwork, which is driven by a weight and controlled by a fan governor. If an anomaly appears in a determined error, any one of these elements may be, and from time to time is, presumably at fault, either accidentally or systematically; for instance, it will appear below that the determinations are systematically different by an amount ranging up to and beyond a hundredth of a second, or, as it will be convenient to name it, to 10 *mil-seconds* (ms.), according to which of the thirty teeth of the scape wheel it is taken from, while three or four times that amount may be introduced by varying the adjustment of the relay.

Attention has of late been directed at the Royal Observatory, Edinburgh, to the improvement of time-keeping, and consideration of the position indicated above showed that little convincing advance could be made without a more direct as well as a more refined method of registration. Chronographs are in existence which are said to record the time to 1 ms., but so far as I know without any sufficient guarantee of their performance. One naturally turns to photography. I believe that the late Professor F. L. O. Wadsworth had a project of taking time by photographing the position of a pendulum at the moment; whether any steps were taken to carry it into practice I do not know, but it would evidently involve difficulties of its own, the chief being a true theory of the motion of the pendulum, and the design of a camera which would operate without introducing a variable lag. A more direct method appeared to offer in the registration of the signal current sent out by the clock itself by means of an oscillograph or kindred instrument. It is true that in only one of the clocks referred to below—Cottingham (C.), with a version of Gill's escapement—is the signal given directly by the pendulum; but in Riefler (R.) and in the synchronome clock (Σ.) the signal is the most direct external message that the clock issues, and the remaining sources of irregularity which they contain are so few in number—being irregularity of the scape wheel in R., the pendulum off beat in Σ., besides accidental variations due to set of the moving parts—that they can be studied without obscurity and their causes and amounts assigned. When this is done the signal current becomes a true and absolute signal from the pendulum. That such a current can be registered by an oscillograph without

introducing any variable extraneous lag whatsoever is quite well known, and it is only necessary to adapt the method to the present case. The oscillograph, in essential, is an Einthoven galvanometer, in which the wire which carries the current to be recorded passes in a loop up and down between the poles of an external electromagnet. When the current passes, the magnetic field causes the two branches of the loop to deviate in opposite directions, and if they are spanned by a mirror, this mirror turns and moves a spot of light which can be used for registration. The fact that the magnetic field is external gives its power to the device; for, no matter how feeble the signal current may be, it can be made to yield a sensible mechanical effect by increasing the strength of the magnetic field, and the same resource permits the leads of the current between the poles of the magnet to be shortened and tightened until the time taken for the wire to assume a new position is reduced to any assigned smallness. Under these circumstances its movement is quite dead-beat under air resistance, and requires no further damping; and, moreover, the time taken for a movement of any amplitude is the same. But in any case the amount of this time is eliminated by the method of measurement, as will be seen below.

We possess at the Royal Observatory a remarkable electromagnet built by Lord Crawford about 1870; no doubt its design would not be repeated now, but it is still a powerful and convenient instrument; the poles are square flats, 5 inches a side (pierced by a central circular hole about 1 inch diameter). These poles can be made to approach or recede from one another, and a field of upwards of 12,000 gaussses per sq. cm. has been measured between them. This instrument forms the basis for the microchronograph which I shall now describe. The loop which carries the signal current is made of platinoid wire, the two sides being $\frac{1}{8}$ inch apart, and under a joint tension of 8 lbs., which is sufficiently short of the point at which the elasticity of the wire gives way; the length of span of the loop is $1\frac{1}{2}$ inches; as regards the mirror, I have had to be content with what could be obtained at the present time, and it might be improved upon; the time taken by the system to assume a new position is about 3 mil-seconds, but, as remarked above, this does not come into the measures.

The general arrangement of the microchronograph is shown in plan in the figure. M, M are the poles of the great electromagnet; *m* is a small flat mirror mounted across the up-and-down wires, *l*, that lead the signal current. A is an arc lantern, focussed upon a slit S. IJ is an interruptor, to be described more particularly below. L is a lens, which with the intervention of the mirror, *m*, throws an image of S, magnified about 4 times, upon the camera, C, where it is received by a cylindrical lens, *c*, which focuses it upon the film, *f*. *f* is a cinema film, perforated and wound off a drum by means of a sprocket wheel, actuated by the gear G, which is driven by a band and small electric motor. A variety of speeds can be given to the film, those most commonly employed being

from 3 to 4 cm. per second. So long as no change of current passes in l , the spot of light traces a straight line on the film. When the clock or other signal makes a change, the spot is deviated. In order then to measure correctly the time of any given signal in terms of the clock signal, it is requisite to interpolate correctly between two successive marks made by the clock. For this the motor cannot be trusted; its audible note rises and falls sensibly within the second. It is therefore necessary to mark out the second into correct subdivisions which shall be short enough to make any acceleration of the motor within them negligible; that is to say, incapable of introducing an error which shall not be well below 1 ms. This is the purpose of the interruptor, IJ. It is a straight steel tongue, securely clamped at J and loaded at I till its semi-period is near $0^{\circ}.1$. It is below the level of the slit, but carries a steel wire, z , projecting from it, almost in contact with

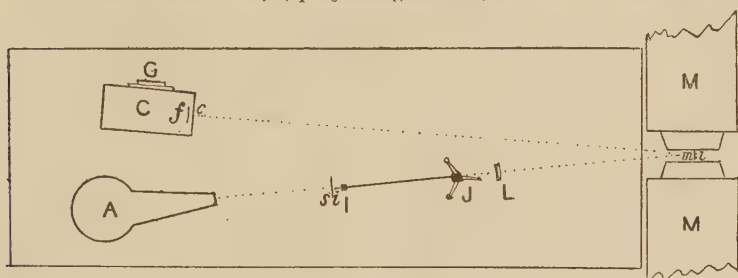


FIG. 1.

the slit, of about the same breadth with it, and occulting it at each semi-oscillation. The tongue IJ is not actuated by any maintaining device. It is simply held aside to a measured degree, and released 20 seconds before the film is exposed. Its oscillations die down naturally in amplitude, the occultations grow broader and become slightly accelerated if the exposure is continued for, say, a minute, but there is nothing of the kind to be noticed in them for an ordinary exposure which lasts for about 15 seconds. The loading of the interruptor is such that the divisions are intermediate in length between the tenths of solar and sidereal seconds, or thereabout. The period is very nearly but not quite the same from one occasion to another; it is convenient to have a name for it that does not suggest that it is an exact subdivision of the second or constant in value, neither of which is requisite for its correct use, and I call it a *decim*.

The film is measured under a travelling micrometer microscope, in terms of the decim and interpolated fractions of the same; backlash is eliminated as usual, by setting on each mark first for the right and then for the left and taking the mean; at the same time the period taken by the mirror to assume a fresh position is eliminated by making the first setting on the end and the second setting on the beginning of its motion. With any except a very

defective trace the work is easy and certain to $\cdot 01$ mm. Provided the *decim* is constant in time for this experiment, and the run of the motor is sufficiently steady over this one *decim*, this is equivalent to a measurement to $0\cdot 3$ ms. or less. The factors that enter then into consideration are the constancy of the period of the interruptor for each occasion, and the uniformity of run of the motor for any single *decim*. It requires then to be established that the faults of these have been reduced to a point where they cannot introduce an error amounting to 1 ms. To this I now proceed, the standard for comparison being the clock Cottingham, which makes and breaks its current with the pendulum itself. Presumably then its 2-second signals are equal much within 1 ms., and if, as will be shown, in each experiment, the measure of Cottingham's seconds in decims is completely steady, I take it as a demonstration both that Cottingham's seconds *are* so equal (which cannot be said of the other clocks) and that the decims are equal too. It will then remain to examine the linear measures of the decims upon the trace, in order to study the acceleration of the motor and show that it is cut up into sufficiently small pieces to make its noxious effects negligible; or, in other words, to prove that a linear interpolation is admissible within a single *decim*. This is due and proper as a direct test, but in substance it is effectively anticipated by the comparison of Cottingham's seconds with the *decim*, which is made through the linear measure of the trace, and which would therefore exhibit any faults arising from interpolation from the latter.

In Table I. are shown for the first twenty traces which have Cottingham's seconds upon them the measures of these in terms of the *decim*. One faint trace that could not be measured accurately has been omitted. Beneath each set are shown numbers completely smoothed, as if calculated from a formula. The total discrepancies are given on the right; for 120 comparisons they amount to 26 units. That is to say, including all errors of record, of measurement, as well as irregularity in the interruptor which marks the *decim* or of Cottingham's second, for a single record on the trace the mean discrepancy from complete smoothness is $\pm 0\cdot 22$ ms. As the method professes to measure to 1 ms., this is well below what we need consider. The conclusion is that, in spite of any faults in the motor or the sprocket drive of the film, the method of measuring from the *decim*-marks and interpolating between them gives a completely reliable result to the thousandth of a second, so that any irregularities that are found when other clocks or signals record themselves must be attributed to these signals themselves, and are not introduced by the microchronograph. The majority of the measures given below were made by Miss K. A. Williams.

TABLE I.

Cottingham's Two-second Intervals in Terms of the Decim.

Trace.	1 ^s .	3 ^s .	5 ^s .	7 ^s .	9 ^s .	11 ^s .	Total Discrepancy.	Measurer.
No. 24	^{0d} ·+ '11	^{20d} ·+ '14	^{40d} ·+ '17	^{60d} ·+ '21	^{80d} ·+ '25	^{100d} ·+ '28	...	R. A. S.
Cf.	['11	'14	'18	'21	'25	'28]	1	
25	'11	'14	'17	'19	'23	'26	...	R. A. S.
Cf.	['11	'14	'17	'20	'23	'26]	1	
26	'88	'91	'94	'97	1'00	1'03	...	R. A. S.
Cf.	['88	'91	'94	'97	1'00	1'03]	0	
27	'30	'34	'36	'39	'43	'45	...	R. A. S.
Cf.	['30	'33	'36	'39	'42	'45]	2	
28	'15	'19	'22	'24	'27	...	...	K. A. W.
Cf.	['16	'19	'22	'24	'27	]	1	
31	'61	'64	'69	'71	'76	'77	...	K. A. W.
Cf.	['61	'64	'68	'71	'74	'77]	3	
32	'64	'68	'71	'74	'77	'80	...	K. A. W.
Cf.	['64	'68	'71	'74	'77	'80]	0	
33	'12	'15	'19	'21	'25	'27	...	K. A. W.
Cf.	['12	'15	'18	'21	'24	'27]	2	
34	'92	'96	1'00	1'00	1'04	1'07	...	K. A. W.
Cf.	['92	'95	'98	1'01	1'04	1'07]	4	
35	'82	'85	'87	'90	'92	'95	...	K. A. W.
Cf.	['82	'85	'87	'90	'92	'95]	0	
36	'45	'48	'51	'53	'56	'58	...	K. A. W.
Cf.	['45	'48	'51	'53	'56	'58]	0	
38	1'73	1'74	1'76	1'79	1'81	1'84	...	R. A. S.
Cf.	['72	'74	'76	'79	'81	'84]	1	
41	2'71	2'73	2'75	2'78	2'80	2'81	...	K. A. W.
Cf.	['71	'73	'75	'78	'80	'82]	1	
42	3'13	3'15	3'18	3'21	3'23	3'25	...	K. A. W.
Cf.	['13	'15	'18	'21	'23	'26]	1	
43	2'84	2'86	2'88	2'91	2'91	2'94	...	K. A. W.
Cf.	['84	'86	'88	'90	'92	'94]		
44	3'09	3'11	3'13	3'15	3'18	3'21	...	K. A. W.
Cf.	['09	'11	'13	'15	'18	'20]	1	
45	4'28	4'28	4'30	4'33	4'36	4'37	...	R. A. S.
Cf.	['28	'29	'31	'33	'35	'37]	3	
46	4'16	4'16	4'19	4'21	4'22	4'24	...	K. A. W.
Cf.	['16	'17	'19	'21	'22	'24]	1	
47	4'33	4'37	4'39	4'41	4'44	4'46	...	K. A. W.
Cf.	['34	'37	'39	'41	'44	'46]	1	
48	4'48	4'50	4'53	4'54	4'56	4'58	...	K. A. W.
Cf.	['48	'50	'52	'54	'56	'58]	1	

Consider now the absolute value of the decim in any operation. Taking the span of 10 seconds of mean time, its value appears as slightly over 100 decims. We get from the successive traces the

values recorded in Table II. Between 69 and 71 there was a break of about 0.6 ms., for which I have no recorded reason to give; presumably there was some disturbance. Thereafter the decim remains very constantly near the same value, but with a slight though distinct tendency to increase in duration. The smaller variations, from one experiment to the next, run to a thirtieth of a mil-second or less. These must be regarded at present as inevitably incident, and are due perhaps to slight changes in the rigidity of the support.

TABLE II.
Value of Decim in Seconds of Mean Time.

Trace No.	Decim = 0.1 - s	Trace No.	Decim = 0.1 - s	Trace No.	Decim = 0.1 - s	Trace No.	Decim = 0.1 - s
24	.0017	69 a	.0010	81 b	.0062	90 d	.0058
25	15	b	10	c	61	e	60
26	15	c	8	d	60	91 a	57
27	15	71 a	64	e	62	b	57
31	16	b	65	82 a	61	c	60
32	16	c	67	b	60	d	57
33	15	72 a	68	c	59	92 a	56
34	15	b	67	e	60	b	57
35	13	c	70	83 a	61	c	58
36	13	74 a	70	b	59	d	58
38	11	b	70	c	60	e	56
41	10	c	69	d	60	93 a	55
42	12	75 c	66	e	58	b	55
43	10	76 b	70	84 e	62	c	55
44	12	c	69	85 a	54	d	53
45	9	77 a	67	86 a	56	e	54
46	8	b	67	87 a	57	94 a	55
47	13	c	67	b	56	b	52
48	10	g	64	c	58	95 b	52
50	9	78 a	66	d	56	c	52
51	8	b	69	e	57	e	54
52	8	c	70	88 a	54	96 a	52
53	12	d	66	b	56	e	51
54	12	e	66	c	56	97 a	50
55	11	f	66	d	57	e	50
56	10	g	67	e	59	98 a	53
57	11	79 b	63	89 a	59	b	50
58	11	c	66	b	58	c	50
59	11	d	66	c	58	d	50
60	10	80 a	63	d	59	99 b	45
61	12	b	64	e	59	d	47
62	14	c	57	90 a	60	e	45
63	13	e	64	b	57	100 a	46
64	12	81 a	61	c	59	f	48

Note.—From 69 onwards several traces, distinguished as *a*, *b*, . . ., were taken in succession on the same film.

As the oscillations of the interruptor die down and its arc diminishes, its period may shorten. An analysis of the traces upon which Table II. is based shows a sharp separation between Nos. 69c and 71a. In the first part no trace of an acceleration is shown; the mean value of the intervals 1^s-3^s and 11^s-13^s , measured in decims, is exactly the same, viz. $20^d.025$. In the second part there is a smooth and well-marked progression.

TABLE III.

Values in Decims of Cottingham's Successive Two-second Interval, derived from Traces 71a to 80e.

s s	d	d
1-3	$20^d.126$	Cf. Calc. $20^d.125$
3-5	$.126$	$.127$
5-7	$.130$	$.130$
7-9	$.133$	$.134$
9-11	$.139$	$.137$
11-13	$.141$	$.141$
13-15	$.147$	$.144$

The column added for comparison is calculated with a factor for progression of length of the decim $1 - .00075$ per 10 sec. Both the change in length of the decim after Trace No. 69 and the sudden appearance of this acceleration are obscure in origin, but are probably due to changes in the fixture of the interruptor. In some earlier work an acceleration had made itself apparent, but of less amount, giving a factor $1 - .0003$ per 10 sec. In later traces again, from No. 86a to No. 93e, it is much reduced, showing a factor $1 - .00015$, the mean values for the intervals 1^s-3^s and 11^s-13^s being $20^d.113$ and $20^d.116$ respectively. In the latest traces it seems to have disappeared completely.

It is of interest now to examine the actual running of the film lineally, taking it as established by the foregoing comparisons that the decims represent known time intervals. We may anticipate variations from three causes: (1) if the vibrating tongue which causes the interruptions is out of centre, the vibrations will be "off beat," and alternate ones will be of different lengths, a difference that will grow more pronounced as the vibration dies away; (2) the lineal measure of the decim on the film will fluctuate with the speed of the motor; and (3) it may be supposed that the sprocket drive on the perforations of the film will move it forward with a certain proportion of jumps and hesitations. These faults may be examined by measuring in millimetres all the decims on the trace. This is somewhat laborious; it was done at the beginning in a considerable number of cases until the character of the motion became clear. Since it is only necessary to show that the faults in question are not present in a degree that will prejudice the accuracy aimed at, once that was made clear, com-

plete measurement was abandoned, and the regular measure now consists of each signal, and the two extremities of the decim that contains it, with perhaps a few others as specimens. As an average illustration I have copied out below the complete measure of Trace No. 29. The measures of the decims are given in millimetres alternately, under the columns A, B. In perfect adjustment A and B should be equal. They are set as nearly equal as can be judged, by bringing the interruptor to rest, and seeing that its shadow, which just fills the object glass, falls correctly upon it. This is easily secured, and, as it appears, is sufficient. It is assumed that in calculating we need not distinguish the series A from series B in length. For this to be justified, consider the effect of a residual error such that the ratio $A : B = 1 + a : 1 - a$.

Measuring from middle to middle we should have equal spaces, and if we calculate as if the decims were equal, the actual interruption itself would be out of truth by $\pm \frac{1}{2}a$, according as we reckon it as belonging to series A or series B. Provided therefore a does not exceed $\cdot 01$, an error of 1 ms. can only be introduced by this calculation if another accidental error happens to conspire with it. To examine the value of a the column C has been formed, being equal to $\frac{1}{2}(A_{-1} + A_{+1}) - B_0$. In this column positive values are set to the left and negative values to the right. The breaks in this column show to what degree the film is moved in jerks. Fluctuations in the drive may be seen by taking the average value of the decims as we proceed down the series A or B; they are, however, of no significance apart from their effect in column C for they will not otherwise disturb linear proportionality.

The mean values of the column C run

	mm	
1st quarter	+ '054	$\frac{1}{2}a = \cdot 003$
2nd „	+ '045	'003
3rd „	+ '068	'004
4th „	+ '108	'007

the increase being due to damping of the vibration. The corresponding values of $\frac{1}{2}a$ are given alongside. From these it is apparent that, systematically, a is well below the prescribed limit ($\cdot 01$). As regards individual values for a , there are 9 cases out of 86 in which a reaches or exceeds the value $\cdot 01$, the largest being $a = +\cdot 015$. It is apparent therefore that the detailed measurement of this trace would lead us to expect fully the prescribed degree of accuracy in time measures which has already been established by comparison of C and the decim. Other traces, of which a large number were measured in detail, led to exactly the same conclusion. It is unnecessary to give the actual figures for them.

TABLE IV.

Trace No. 29. Linear Measures of Decim.

A.	B.	C.	A.	B.	C.	A.	B.	C.
mm	mm		mm	mm		mm	mm	
3'40	3'35	+ '07	3'73	3'77	-7	3'78	3'68	+ 8
'43	'42	'00	'67	'67	+ 5	'73	'68	6
'41	'53	- '05	'76	'60	19	'75	'74	5
'55	'48	12	'81	'72	2	'82	'76	5
'65	'53	12	'66	'77	- 12	'80	'63	16
'64	'63	1	'63	'62	9	'77	'82	- 8
'63	'62	3	'79	'62	11	'70	'66	12
'67	'70	- 2	'67	'69	- 4	'85	'63	21
'69	'61	11	'63	'59	12	'82	'68	12
'74	'57	20	'79	'60	10	'78	'60	18
'80	'62	12	'61	'67	- 01	'77	'67	8
'68	'70	- 2	'71	'63	10	'73	'72	- 5
'68	'74	0	'75	'63	11	'61	'73	- 2
'80	'67	14	'73	'64	10	'80	'60	22
'82	'67	10	'75	'68	5	'83	'73	6
'72	'74	- 1	'70	'65	6	'75	'69	12
'73	'68	4	'72	'68	9	'87	'64	19
'70	'73	0	'82	'66	15	'79	'70	5
'76	'73	6	'80	'71	2	'70	'76	- 2
'81	'77	4	'66	'75	- 04	'77	'70	11
'80	'73	7	'76	'74	6	'84	'63	23
'80	'70	6	'84	'66	12	'88	'71	11
'71	'80	- 8	'72	'79	- 6	'75	'69	10
'73	'79	- 2	'73	'73	5	'82	'71	10
'81	'74	10	'83	'68	16	'79	'73	5
'86	'70	7	'85	'76	10	'76	'74	5
'68	'82	- 10	'87	'73	11	'81	'70	11
'76	'71	11	'81	'75	4	'82	'68	+ 12
'87	'70	10	'77	'72	6	3'77	3'73	
3'73	3'77	-7	3'78	3'68	+ 8			

Having completed these preliminaries, we can now examine the anomalies shown in the signals sent out by the clocks Σ . and R., in which the signal current is not taken directly from the pendulum. Take first the clock Σ . This clock consists of a pendulum and maintenance merely — the counting train being external and subsidiary, and operated by the signal current. The maintenance is a stroke alternately on one side and the other of a crutch that

is linked to the pendulum. The pendulum unlocks the striking piece, and when the stroke is made the striking piece continues to fall and releases a winding piece which resets the striking piece in preparation for the return of the pendulum; the winding piece moves on and makes a contact which serves as signal current and also resets the winding piece.

It may be expected that the several operations separating the signal from the pendulum itself will appear in the form of an increased accidental error of the signals recorded. Besides this error of a purely accidental character, there may be a systematic difference between alternate seconds owing to the pendulum being off beat. Table V. shows the result of measuring five traces. The numbers shown are *Obs.-Calc.*, expressed in mil-seconds. The calculation was made by comparing the observed measures in decims with a series proceeding by constant differences. The even seconds and odd seconds were then compared for indications of systematic difference. For Traces Nos. 29 and 31 there was a difference of 2 ms., for the rest zero. This difference was removed, and the residuals shown in the table are the accidental effect of the method of sending the signal. The general mean for 82 measures is ± 1.1 ms., which may be compared with ± 0.2 ms. for C. The greatest individual discrepancy is 3 ms. As the discrepancies are accidental, they must be removed by taking the mean of a sufficient number of comparisons.

As Cottingham is a mean time clock and synchronome a sidereal one, it is of interest to compare the ratios of their seconds to one another as taken from these traces over a ten-second run in each case. We find the following values, with which may be compared the true ratio 1.00273.

Trace No.	Σ d	C. d	Ratio.
31	99.91	100.16	1.0025
32	.90	.17	27
33	.84	.14	30
34	.87	13	26
Mean			1.0027

TABLE V.
Anomalies in Seconds of Clock Σ.

Trace No.	29.	31.	32.	33.	34.
sec	ms	ms	ms	ms	ms
1	+2	+2	-2	0	0
2	+1	+2	+2	-1	0
3	-1	0	+2	+3	0
4	+1	-1	-1	-2	+1
5	-1	0	*	+2	0
6	0	-1	0	0	-2
7	-1	-1	0	0	0
8	-2	0	+1	0	+1
9	+3	0	-1	0	+3
10	-2	0	+1	+1	-1
11	+1	-1	-1	+3	+1
12	0	+3	-1	0	0
13	-1	0	+3	+2	0
14	+2	0	-1	0	+1
15	-1	-1	-3	-1	0
16	-1	-1	-2	+3	
17	-1	...	2		
18	0	...	+3		
Means	$\pm 1'2_{18}$	$\pm 0'8_{16}$	$\pm 1'5_{17}$	$\pm 1'1_{16}$	$\pm 0'7_{15}$

General mean $_{82} \pm 1'1$ ms.

I take next the signals of the clock Riefler. The signals here also are indirect. The clock possesses a dial and wheel train. The driving weight actuates the "third wheel" which engages with the scape wheel. The scape wheel engages the anchor which maintains the pendulum in motion by bending its suspension spring. The scape wheel has 30 teeth, as usual. On the scape wheel arbor another wheel is mounted, possessing also 30 sloped teeth—less one, cut away to mark the beginning of the minute. These teeth alternately lift and let fall a pivoted arm, and thereby break and make the contact for the signal. We may expect, then, two sources of irregularity in the signals: first, a systematic one due to differences in the teeth of the scape wheel, and the second, more or less accidental, due to "set" of these various pieces in slightly different places allowed them by the "shake" they have to possess. The question of beat does not enter, because only the break is measured, and, as in Cottingham, this occurs only every two seconds.

The positions of the teeth of the scape wheel, with respect to one another, are of course very important, for, as it will appear,

some of them are out of relative place by more than 10 ms., and in consequence no consistent micro-measures can be made unless the positions are found and the second with which any other signal is compared is known and its error applied. The determination of their positions was made twice over, first, in a preliminary way with paper traces, and secondly, for use, with films. I shall deal with the latter first.

The determination is made by reading Riefler and Cottingham simultaneously upon the trace; then Riefler's two-second intervals are measured in terms of the decim and the decim converted by reference to Cottingham's two-second intervals. It is proposed, in use, generally speaking, in comparing any set of signals with Riefler, to employ a run of, say, 10 or 12 secs. of Riefler, beginning from 1^s and going to 11^s or 13^s, and therefore this section has been done with more fulness than the rest. The results are shown in Table VI. When the mean of each column is taken and the residues derived, the mean of 244 residues is ± 1.1 ms., exactly equal to the mean accidental discrepancy of a signal of Σ . The outcome of the table shows that if Riefler's signal at 1 sec. be taken as the standard, a comparison with the other seconds may be reduced to it by applying to the measures: for

s	ms	s	ms
3	- 7	9	- 12
5	- 11	11	- 11
7	- 9	13	- 3

where a negative sign indicates that the interval from 1 sec. to the second in question is too short.

TABLE VI.

Clock Riefler. Value of Successive Two-second Intervals.

Trace. No.	1 ^s -3 ^s .		3 ^s -5 ^s .	5 ^s -7 ^s .	7 ^s -9 ^s .	9 ^s -11 ^s .	11 ^s -13 ^s .
	s	ms	ms	ms	ms	ms	ms
35	2	- 1	-7	+4	-2	+1	*
36		- 8	-5	+3	-2	+1	+ 8
37		- 5	-8	+1	0	+1	*
38		- 4	-5	+2	-1	+2	+ 7
41		- 7	-6	+2	-1	+1	+ 7
42		- 5	-3	+1	-2	+2	+ 8
43		- 4	-5	+1	0	+2	+ 6
44		- 5	-4	+2	-2	+1	+ 5
45		- 6	-6	+2	-1	+3	+ 5
46		- 7	-4	+2	-4	0	+10
47		- 4	-5	+3	-4	+2	+ 5
48		- 7	-3	+2	-3	+1	+ 5

TABLE VI.—*continued.*

Trace.	1 ^s -3 ^s .		3 ^s -5 ^s .		5 ^s -7 ^s .		7 ^s -9 ^s .		9 ^s -11 ^s .		11 ^s -13 ^s .	
	s	ms	ms	ms	ms	ms	ms	ms	ms	ms	ms	ms
No. 69 A	-	7	-4	+2	-2	0		+7				
B	-	7	-5	+2	-2	-1		+6				
C	-	7	-4	+2	-4	+2		+6				
71 A	-	5	-7	+3	-3	+2		+9				
B	-	8	-5	+3	-4	+1		+8				
C	-	7	-4	+2	-2	0		+10				
72 B	-	7	-4	+3	-2	+1		+11				
C	-	7	-5	+2	-2	0		+10				
74 A	-	7	-5	+3	-5	0		+9				
B	-	6	-6	+4	-7	+2		+8				
C	-	6	-5	0	-3	+2		+8				
76 B	-	8	-6	0	0	-2		+10				
C	-	10	-2	-2	-3	+3		+5				
77 A	-	5	-4	+1	-3	+1		+8				
C	*		-4	+1	*	*		+7				
G	-	9	-2	+1	-4	+1		+9				
78 A	-	6	-6	+2	-2	+1		+9				
B	-	5	-6	+1	-3	+1		+8				
C	-	7	-4	+4	-6	+4		+6				
D	-	4	-6	0	-1	+2		+8				
E	-	8	-5	+3	-3	+3		+7				
F	-	8	-3	+2	-3	+2		+8				
G	-	8	-1	0	-1	+2		+8				
79 B	*		*	+2	-5	+1		+9				
C	-	5	-5	+2	-5	+1		+8				
D	*		-5	+2	-4	+1		+9				
80 A	-	9	-5	+1	-3	0		+8				
B	-	9	-3	+1	-3	+1		+9				
C	-	4	-4	0	-3	+2		+9				
E	-	11	-4	0	-2	+2		+5				
Means	-6, 5 ₈₉		-4, 6 ₄₁	+1, 7 ₄₂	-2, 7 ₄₁	+1, 3 ₄₁		+7, 7 ₄₀				

Table VI. is continued by Table VII., which gives the corresponding intervals as determined for the remainder of the wheel. For this purpose the 16 traces Nos. 49-64 were taken expressly and the results collected in the same way. The excesses or defects of the two-second intervals in mil-seconds are given under the column *mean*.

TABLE VII.

Riefler's Escape Wheel. Value of Successive Two-second Intervals—continued.

No. 49.				No. 53.		No. 57.		No. 61.		Mean.	
s	s	s	ms	s	ms	s	ms	s	ms	s	ms
11-13	2	-		2	+9	2		2	+7		+8
13-15		- 2		0		+2		+4			+1
15-17		0		+2		+2		+1			+1
17-19		- 6		-6		-7		-9			-7
19-21		- 1		1		0		+2			0
21-23				0		+1		-1		No. 62.	0
										s	ms
23-25			No. 50.	+6	No. 54.	+2	No. 58.			2	+3
			s	ms	ms		s	ms			
25-27			2	-4	-2	-6	0	2	-1	0	-2
27-29			+4		+3		+4			+2	+3
29-31			-7		-8		-5			-7	-7
31-33			+8		+11		+8			+7	+8
33-35			-2		-2		-7	No. 63.		-2	-3
35-37	No. 51.	+8	No. 55.	+7	No. 59.	+10	+8			+9	+8
37-39	- 2	-1	-2	0	-3	-4	-1				-2
39-41	+7		+8		+3		+5				+6
41-43	-10		-4		-3		-6				-6
43-45	-1		-7		-4		-3				-4
45-47	+6		+6	No. 56.	+5	No. 60.	+5	No. 64.		+5	+5
47-49	-5	No. 52.	-1	-4	-2	-1	-6			-5	-4
49-51	0	-2	-4	-2	-5	-3				-3	-3
51-53		+4		+4		+3				+5	+4
53-55		-3		+1		-1				-3	-2
55-57		2	-1	+3		+1				+4	+2
57-1		4	+3	+2		0				+3	+2

A preliminary determination of these intervals had previously been made upon paper traces, and the final determination just recorded was intended to confirm it; but an accident occurred between the taking of the two sets, by which Riefler's current was short circuited for an instant, and the fine helical wire leading the current to the insulated pivoted arm was burnt out. The clock was opened and the wire was replaced by one that may have been slightly stronger, no other change being made within the clock. When the new determination was made, it appeared on comparison that the adjustment had undergone a systematic change. The values of the preliminary and final corrections from any given second to 1 sec. as standard are shown in Table VIII. The preliminary determination need not be given in detail; it is little inferior to the final one, and the changes are unquestionably real. The mean change has been reduced to zero by applying +2, 5 ms. to each; that is to say, by considering the one-second point to be altered by that amount.

TABLE VIII.

Riefler's Wheel. Reductions to 1 Second and Change consequent on Adjustment.

	Final. ms	Prelim. ms	Change. ms		Final. ms	Prelim. ms	Change. ms
s							
3	- 7	- 2	- 2	33 ^s	- 3	0	- 1
5	- 11	- 9	0	35	- 6	- 1	- 2
7	- 9	- 4	- 2	37	+ 2	+ 4	0
9	- 12	- 7	- 3	39	0	+ 3	0
11	- 11	- 8	0	41	+ 6	+ 5	+ 3
13	- 3	- 2	+ 1	43	0	+ 2	+ 1
15	- 2	- 3	+ 4	45	- 4	+ 1	- 3
17	- 1	- 3	+ 4	47	+ 1	+ 3	+ 1
19	- 8	- 7	+ 2	49	- 3	0	- 1
21	- 8	- 6	0	51	- 6	- 3	0
23	- 8	- 8	+ 3	53	- 2	+ 1	- 1
25	- 5	- 6	+ 3	55	- 4	- 3	+ 2
27	- 7	- 2	- 2	57	- 2	+ 1	- 1
29	- 4	+ 1	- 3	I	0	...	+ 3
31	- 11	- 5	- 3				

The change is presumably due to some difference in set of the pivoted arm with respect to the toothed wheel that actuates it. The example may be taken as demonstrating how undesirable is a train of toothed wheels in a precision clock, particularly in connection with sending out signals. We very soon revert to the difficulty remarked at the beginning of this paper, in which it would be impossible to fix the responsibility for the occurrence of sensibly erratic signals, since they are dependent upon a number of adjustments and upon accidental set.

I shall next give some examples of the use of the microchronograph, and first for determining certain lags between signals originating from the same source.

Lag of the Fuess Chronograph.—This is the chronograph we use for general purposes. It is sufficient here to say that it carries three prickers, mounted side by side, and actuated by separate electromagnets. These make imprints upon a tape of paper which is carried past them by clockwork. In normal working the clock R. operates the first pricker, the sidereal clock Σ. operates the middle one, and the mean time clock C., or the transit circle signal, operates the third. A current of about 0.3 amp. is required to operate these prickers, so that the clocks have to work them by means of relays. It is required to find the amount of lag between the true clock current and the perforation of the paper tape.

The clocks and the chronograph are on different floors, and are separated by the whole length of the buildings, but it is a simple matter to arrange that the pricker shall close a circuit when it makes its perforation and to lead the resulting current to the microchronograph, where it records itself alongside the original clock signal.

The results show that the lag is remarkably consistent, not

only throughout a single experiment, but from one experiment to another, provided no readjustment of the pens or relays is made; but the lags are liable to differ for the different pens, and are dependent upon adjustment, presumably also upon the current through the electromagnets, the range of alteration amounting to several hundredths of a second. The accidental variations of the clocks, and for the clock R. the inequalities of the scape wheel, are reproduced with fidelity by the chronograph.

Take, for example, the traces 75 *a, b, c*. Denote the Fuess chronograph signal by X, and its lag upon the clock by X_R , X_C , or X_Σ .

TABLE IX.

Lag of Fuess Chronograph (FX) in Successive Seconds.

No. 75a.			No. 75b.			No. 75c.		
R.	FX.	X_R .	Σ .	FX.	X_Σ .	C.	FX.	X_C .
d	d	ms	d	d	ms	d	d	ms
0'61	1'14	53	0'40	1'34	94	0'81	1'38	57
20'66	21'19	53	10'47	11'40	93	20'94	21'50	56
40'73	41'27	54	20'48	21'42	94	41'06	41'60	54
60'89	61'44	55	30'54	31'47	93	61'20	61'75	55
81'00	81'55	55	40'57	41'50	93	81'33	81'87	54
101'15	101'70	55	50'61	51'56	95	101'45	102'02	57
121'37	121'92	55	60'65	61'57	92	121'60	122'15	55
	Mean	54	70'71	71'64	93	141'75	142'31	56
			80'74	81'65	91		Mean	56
			90'80	91'73	93			
			100'84	101'76	92			
				Mean	93			

It will be remarked that the seconds of R. are noticeably unequal and that Σ . is slightly off beat, and that the chronograph, as we might expect, reproduces these inequalities.

The following are records of the mean lag for different experiments determined as above:—

TABLE X.

Lag of Fuess Chronograph on Primary Signals.

No. of Trace.	X_R . ms	No. of Trace.	X_Σ . ms	No. of Trace.	X_C . ms
75a	54	75b	93	75c	56
78a	63	96c	125	95e	46
82c	62	97b	125	96a	47
94c	52	98e	125	97e	46
94d	52	100b	126	98b	46
95a	57	100d	121	98c	46
95d	56	100e	117	100a	45
96b	58	103b	119	100f	46
97d	56	103d	145		
98d	61	104e	62		
99c	52				

A bar marks a change of adjustment of the Fuess system.

Slight adjustments of the relay operated by R. are made from day to day. This no doubt is the reason why the lag X_R is less consistent than C_R . Where no adjustment is made, the lag is surprisingly constant, but it is dependent upon adjustment to the amount of certainly $0^s.03$. Moreover, while the clocks R. and C., which have a similar system and hold the current on and off for whole seconds alternately, do not show much difference in lag, the clock Σ , which gives a short signal lasting about $0^s.05$ out of each second, differs from them sometimes by as much as $0^s.08$.

Besides the Fuess chronograph signal, a signal is taken from the clock R. for comparison with the wireless signals sent from Paris. This is done by means of a controlled clock. The direct current from R. energises in its circuit an electromagnet. A clock, standing in the observatory lobby, carries on its pendulum an armature which swings over and close to this magnet at its extreme excursion on one side, and the clock is thus controlled. For the purpose of sending a signal to the wireless apparatus which shall be as clean and neat as possible, the pendulum in passing its lowest position lifts a short spring which throws a small hammer into momentary contact with a plate. On each occasion the contact is adjusted until it sounds absolutely sharp in the telephone.

As we might expect, this ticker does not reproduce the inequalities of R.'s wheel, and R.'s signals require to be reduced to standard before a smooth comparison can be made; for illustration, consider the trace No. 74c.

TABLE XI.

Trace No. 74c. Lag of the Ticker (T_R) on Riefler—Individual Seconds—before and after Correction for Riefler's Wheel.

	T_R			T_R	
	Uncorrected.	Corrected.		Uncorrected.	Corrected.
	^s	^s		^s	^s
1	+ '147	+ '147	9	+ '158	+ '146
3	'153	'146	11	'157	'146
5	'156	'145	13	'150	'147
7	'154	'145	15	'149	'147

As the contact is out of operation when not in use and is reset on each occasion, the lag T_R must be expected to differ in some degree from one experiment to another. The amount of this variation appears to lie within $\pm 0^s.01$. But in addition, as is well known, the lag of the controlled clock will be dependent upon the controlling current. In normal working this current amounts to 10 milliamps. To examine this point, the current was reduced from 10 milliamps to 6 milliamps, and then raised again, leaving in each case intervals of about an hour for the clock to pick up its new position. The following table shows the collected results:—

TABLE XII.

Lag of Ticker (T_R) on Riefler; Successive Experiments.

No. of Trace.	T_R s	No. of Trace.	T_R s	No. of Trace.	T_R s
68	+ '168	71b	+ '160	74a	+ '155
69a	'164	71c	'154	*74b	'172
69b	'160	72a	'159	74c	'146
69c	'160	*72b	'193	99e	'145
71a	'157	72c	'159		

* Controlling current reduced to 6 milliamps.

As the indications of the Fuess chronograph have been adopted hitherto as the time given by Riefler, when its error is found with the transit circle, we see by comparison with Table X., column X_R , that the ticker system for comparison with the wireless signals is slow by 0^s.10. This lag had previously been determined by ear by the help of an intermediate free pendulum which gained about one beat per minute upon the seconds ticks. The value adopted was 0^s.08; but from a variety of circumstances it was rather troublesome to find, and the determination might well be 0^s.02 in error.

I shall give next some examples of determination of the relative error of two clocks. Three points of interest present themselves: (1) the consistency of independent experiments taken so close together that rate hardly enters; (2) allied to the former, definite indication of irregularities of going which cannot be ascribed to the method of registration; and (3) the value of indications taken with the Fuess chronograph.

It was seen above that the mean of the error of a registered signal of R. or Σ . was ± 1.1 ms., while that of C. was 0.2 ms. Hence the probable error of a single comparison of R. or Σ . with C. is ± 0.95 ms., and of R. with Σ ., ± 1.31 ms. Six or seven comparisons are measured upon the trace, and hence the probable error of a relative time determination from a single trace is not more than 0.4 ms. for C. with R. or Σ ., and 0.5 ms. for R. with Σ . Hence we may expect comparisons made in this way to be rarely ± 1 ms. in error, although naturally one cannot insist on the last unit, for a variety of reasons.

The following examples show the comparison of this expectation with actual results.

Between Nov. 23 and Dec. 2 sets of traces were taken with different parts of R.'s scape wheel in order, primarily, to determine the inequalities of the wheel. They are, however, also available as relative time determinations, if the reduction is applied to bring them to a comparable basis at 1 sec. from Table VIII. Each set consisted of four traces, and there were four sets in all. Three further sets were taken between Dec. 7 and 9, in the standard way between 1 sec. and 13 sec. of R.'s wheel. The results are as follows, no allowance being made for rate except for the last, where the interval between two traces is upwards of an hour.

TABLE XIII.

Determination of Cottingham's Correction relative to Riefler (C_R) at Short Intervals.

No. of Trace.	Time.			C _R .	No. of Trace.	Time.			C _R .
	d	h	m			d	h	m	
49	Nov. 23	22	17	+ '492	61	Dec. 2	21	49	+ '395
50		22	27	'494	62		21	59	'395
51		22	31	'493	63		22	6	'394
52		22	36	'491	64		22	11	'392
53	Nov. 25	21	45	+ '630	69a	Dec. 7	5	31	- '376
54		21	49	'628	b		5	38	'376
55		21	58	'627	c		5	43	'374
56		22	3	'630	71a	Dec. 8	2	42	- '513
57	Nov. 26	21	53	+ '720	b		2	46	'515
58		21	58	'718	c		2	51	'514
59		22	4	'719	72a	Dec. 8	22	24	- '653
60		22	9	'718	b		23	41	'653
					c	9	1	0	'654

If we compare the determinations for one day with the next, it is obvious that the general relative rate of the clocks has been by no means constant. If we consider then that out of the discrepancies of the determinations above we have to provide for actual changes of error as well as of faults of the microchronograph, the numbers seem fairly to confirm the claim that they are a true record to 1 ms., and wherever we find inequalities beyond that order they must be ascribed to actual changes in the clocks.

I now come to the question of the comparative value of times taken by the Fuess chronographs (FX). As mentioned at the beginning, FX is operated with the aid of relays. The great influence of these upon the time recorded was not realised when the experiments began, and their arrangement had been settled more or less arbitrarily. R. operated its relay upon the make of the current, and Σ. and C. upon the break. After Dec. 9, with the intention of putting R. and C. upon the same footing, C.'s relay was made to operate also upon the make. The make for R. is fairly sharp, as may be seen by inspecting the traces, but that of C. is liable to some obscure variation when the loose crutch is picked up by the moving pendulum. The consequence of the change is very conspicuous, both total amount and variation of the equation $FX - MX$ being much increased. When this was seen, both R. and C. were set to operate upon the break, and the discrepancy disappeared. The conclusion is that the indications of FX are now reliable to ± 0.01 , provided always no change is made in the adjustment of the relays or the prickers. But in this connection Table X. should be looked at, and the remarks upon it. By changes of adjustment the readings of FX can be altered by several hundredths of a second, nor can one tell by how much it is altered without the aid of the microchronograph.

TABLE XIV.

Lag of Fuess Chronograph on Microchronograph in comparison of Relative Errors of Clocks C., R.

(1)				(2)				(3)			
Date.		FX-MX.		Date.		FX-MX.		Date.		FX-MX.	
d	h	s		d	h	s		d	h	s	
Nov. 16	9		'00	Dec. 10	9		- '05	Jan. 24	9		'00
	21		'00		21		- '04		21		'00
17	9		- '01	11	9		- '03	25	9		'00
	21		'00		21		+ '07		21		'00
18	9		- '01	12	9		+ '06	26	9		+ '01
	21		+ '01		21		+ '07	27	9		'00
19	9		- '01	...					21		'00
	21		- '02	18	9		+ '05	28	9		- '01
20	9		- '01		21		+ '05		21		'00
	21		- '01	19	9		+ '07	29	9		+ '01
21	9		- '01		21		+ '07		21		- '01
	21		- '01	21	9		+ '10	30	9		'00
22	9		+ '01	22	9		+ '07	31	21		'00
	21		'00		21		+ '10	Feb. 1	9		'00
23	9		- '02	26	9		+ '11		21		+ '01
	21		- '01	31	9		+ '07	3	21		- '01
24	9		'00		21		+ '09	5	9		- '01
	21		- '01	Jan. 1	9		+ '10		21		'00
25	9		+ '02		12		+ '12	6	9		- '02
	21		'00	2	9		+ '10	7	9		'00
26	9		+ '01		12		+ '12		21		+ '01
	21		'00	3	9		+ '06	8	9		'00
Dec. 3	9		'00		12		+ '08	R. and C. operating on break.			
	21		'00	4	9		+ '07				
4	9		- '01		12		+ '07				
	21		- '02	5	9		+ '06				
5	9		- '03		12		+ '06				
	21		- '01	6	9		+ '06				
6	9		- '03		12		+ '08				
	21		- '02	7	9		+ '08				
7	9		- '02		21		+ '09				
	21		- '01	...							
8	9		- '02	21	21		- '01				
	21		+ '02	22	9		+ '06				
9	9		- '02		21		+ '06				
	21		- '01	23	9		+ '06				
R. operating on make				R. and C. operating on							
C. ,, on break				make.							

This table requires no further comment.

The present communication is confined to a description of the microchronograph and to establishing its performance and capa-



FIG. 2.—Drawings from actual Traces. Enlarged about three times.

bilities; I shall not discuss at all the going of the clocks themselves, but shall proceed to describe the illustrative traces shown in fig. 2. These five traces are respectively: (1) clocks Riefler

and synchronome I. ; (2) clocks Cottingham and synchronome II. ; (3) Riefler with controlled ticker ; (4) Riefler with Fuess chronograph ; (5) synchronome I. with Fuess chronograph. The method of sending the signal recorded differs in each case. To take the synchronome clocks first, the signal is made every second by an arm which is thrown into contact by gravity by the release of a click, and is broken again after about 0^s.05 by a spring worked by an electromagnet which the signal itself actuates. The indentation of the line of the trace therefore gives on small scale a graph of the growth of the current. It may be compared with the full set of oscillograms of this current under various circumstances shown by Mr. Hope Jones in his paper "Modern Electric Time Service," *Inst. Elec. Eng. Jour.*, vol. xxix., 1900, p. 119. The essential features are here readily recognisable. After the first contact the pieces rebound mechanically and the current breaks, showing a zigzag in the line. They then lie in contact and the current grows, but not smoothly to its maximum, owing to the entry of self-induction. In Σ . I. the contact pieces are of carbon, in Σ . II. they are of platinum ; a difference in character between the two can be quite well recognised in the traces.

In the clocks Riefler and Cottingham the contacts are made and broken on alternate seconds. Riefler, by a wheel upon the scape wheel arbor, at the odd second, lifts out of contact a freely pivoted arm, and at the even second allows it to fall into contact again. Hence the current is on for a whole second and off for the same time. The mechanical rebound when the arm falls into contact may be seen very clearly and regularly at the make. In Cottingham the make of the current is effected by the pendulum itself striking and carrying with it the crutch. As might be expected, the make is less decided and sharp than in Riefler, but the construction of the clock makes the break very neat and clean.

Little need be said of the three traces showing records by controlled systems. The first shows, along with Riefler's seconds, the tick taken from the pendulum of the controlled lobby clock which is taken to the telephone at the wireless table. As shown above, this is free from the inequalities of Riefler's scape wheel. Its lag on R. can be varied by varying the strength of R's. current. It comes on every second, but its mark is somewhat different according to whether Riefler's own current is off or on. The signals in the last two traces are produced by the pricker arms of the Fuess chronograph, actuated respectively by Riefler and synchronome I. They reproduce any original inequalities of the signals sent by the respective clocks, and, as shown above, their lags, which are here evidently different, can be varied by large amounts by changing the connections of the relays or other details.

*On the Literal Development of the Motion of the
Lunar Perigee.* By R. Moritz.

(Communicated by P. H. Cowell, D.Sc., F.R.S.)

Some years ago, at the suggestion of Dr. Cowell, Engineer-Commander Godbeer, R.N., repeated Hill's well-known development of the variational terms in the motion of the moon.* The very surprising fact was then discovered that these famous series are vitiated by no less than three errors, which have apparently remained undetected for forty years. Dr. Cowell and I both repeated the work, and satisfied ourselves that the facts are as stated. The terms in error are those in m^7 in α_{-3} , in m^8 in α_{-2} , and in m^8 in α_{-3} respectively, and their correct values are given below. Throughout this paper the symbol m refers to Hill's parameter, being $\frac{m}{1-m}$ in terms of that used by the older lunar theorists.

Partly as a result of this work, Dr. Cowell and I embarked upon an investigation of the literal series of characteristic e in the lunar theory, following a method which he had propounded for obtaining the power series expressing that portion of the motion of the perigee which is dependent on the ratio of the mean motions only. The work was afterwards extended to further classes of terms, *e.g.* those depending upon e^2 , e^3 , g , and $\frac{a}{a'}$ and its powers, but the pressure of other duties and interests precluded any idea of the completion of an algebraic lunar theory. It is sufficient to say that all these results, when transformed into M. Andoyer's notation, agree without exception with the full developments which he has given.†

Since the publication of Professor E. W. Brown's Lunar Theory,‡ containing a complete numerical expression for the motion of the perigee carried to a degree of accuracy comparable with modern observations and, indeed, agreeing with the latter to a surprising extent, the question of the development of this quantity in power series has in a sense lost its interest. But though in the case of our own moon no algebraic series can compare in accuracy with this numerical theory, nevertheless the historical interest of the subject, and the possibility of a need for such developments in the case of other satellites, still invest the matter with sufficient importance to warrant this investigation. I ought to state that the inception of the plan and the details of its working are entirely due to Dr. Cowell, who kindly permits me to publish the results.

The history of the expansion in power series of the motion of the perigee is a matter of some interest. Following upon the earlier investigations of Newton and Clairaut, the famous and unexpectedly

* *Collected Works*, vol. i. p. 317.

† *Bulletin Astronomique*, 18, 177 (May 1901) *et seq.*

‡ *Mem. R.A.S.*, 53, 39, 163; 54, 1; 57, 51; 59, 1.

large term $-\frac{2^2 5}{2^5} m^3$ was first given in a literal form by D'Alembert.*

The development was carried on step by step by the intervening workers Plana and de Pontécoulant, but it was not until 1867 that Delaunay † published his value of this quantity. He subsequently ‡ gave, as the result of enormous labours, an expansion of the motion of the perigee to the ninth power of the parameter m .

This was accepted as the last word on the subject until Andoyer,§ by a different method, presented a new value to the same order of approximation differing from Delaunay's result in the terms in m^8 and m^9 . In order to clear up this question; Dr. Hill || undertook to recompute the series by yet another method, utilising the same linear equations which yield his well-known infinite determinant. His process, carried as far as the term in m^{11} , is most elaborate and intricate, and, so far as I can judge, must have involved its author in labours trying even to his robust patience. In the result he confirmed Andoyer's value and rejected that of Delaunay as far as the terms in m^8 and m^9 are concerned. He added two terms in m^{10} and m^{11} , which have stood as correct to the present day. It is the object of this paper to show that the coefficients of m^{10} and m^{11} in Hill's expression are erroneous, and to supply the correct values.

As far as choice of method is concerned, it has been truly said that no man can judge accurately of the relative merits of rival plans in lunar theory until he has made extensive trials of all. It can, however, be stated that any attempt to push the methods of Laplace or de Pontécoulant to powers of m beyond the sixth or seventh is hopeless, while even the industry of a Delaunay might shrink from the idea of any extension on his lines beyond the term in m^9 . The method of Hill certainly led him as far as the term in m^{11} , but it seems to me that this required a power of application which is given to few astronomers. The method used here seems much the least laborious of any hitherto employed, excepting perhaps, the second method of Andoyer, of which I have not made sufficient trial to express an opinion of any value. The expression given below cost us some months of work, it is true, but it was work done in time taken from other and more pressing pursuits. Professor Brown ¶ has remarked that, while it is possible to approximate to c_0 , the quantity in question, during the solution of the equations for ϵ_i , ϵ'_i (the coefficients of the terms of characteristic e), it would be very troublesome. This is, however, the plan adopted.

We have followed the notation of Brown** and made use of the equations which form the basis of his theory. As we were seeking the terms of characteristic e , these took the form

$$\zeta^{-1}(D+m)^2 u_e + M u_e \zeta^{-1} + N S_e \zeta = 0,$$

* *Théorie de la Lune* (1754).

† *Mém. de l'Acad. des Sc.*, 29.

‡ *Comptes Rendus*, 74, 19.

§ *Ann. de la Faculté des Sc. de Toulouse*, tome 6.

|| *Annals of Mathematics, U.S.A.*, 9, 31.

¶ *Lunar Theory*, p. 210.

** *Mem. R.A.S.*, 53, 39.

where

$$M = \frac{1}{2}m^2 + \frac{1}{2}\frac{\kappa}{\rho_0^3},$$

$$N = \frac{3}{2}m^2 + \frac{\frac{3}{2}\kappa u_0^2 \zeta^{-2}}{\rho_0^5},$$

the solution being of the form

$$u_e \zeta^{-1} = a e \sum_i (\epsilon_i' \zeta^{2i+c} + \epsilon_i' \zeta^{2i-c}).$$

We first obtained the corrected values of the a_i , the variational terms computed by Hill.* The process is quite simple, and will not be given here. We then computed the quantities M and N used by Brown, which are obtained from the a_i as follows:—

From the equations of motion

$$\zeta^{-1}(D+m)^2 u_0 + \frac{1}{2}m^2 u_0 \zeta^{-1} + \frac{3}{2}m^2 s_0 \zeta \cdot \zeta^{-2} - \frac{\kappa u_0 \zeta^{-1}}{\rho_0^3} = 0.$$

Put

$$u_0^{-1} \zeta = \Sigma h_i \zeta^{2i} = H,$$

and

$$(2j+1+m)^2 a_j + \frac{1}{2}m^2 a_j + \frac{3}{2}m^2 a_{-j-1} = J_j.$$

Then

$$\frac{\kappa u_0 \zeta^{-1}}{\rho_0^3} = J = \Sigma J_i \zeta^{2i}.$$

Hence

$$\frac{\kappa}{\rho_0^3} = JH,$$

or

$$2M_j^2 = \Sigma J_{j-i} h_i, \text{ when } j \neq 0,$$

and

$$2M_0 = m^2 + \Sigma J_{-i} h_i.$$

Further,

$$\frac{N - \frac{3}{2}m^2 \zeta^{-2}}{\frac{3}{2}} = \frac{\kappa u_0 \zeta^{-1}}{\rho_0^3 s_0 \zeta} = J\bar{H} \text{ (say),}$$

where

$$\bar{H} = \Sigma h_{-i} \zeta^{2i} = s_0^{-1} \zeta^{-1}.$$

Hence

$$\frac{2}{3}N_j = \Sigma J_{j-i} h_{-i}, \text{ when } j \neq -1,$$

and

$$\frac{2}{3}N_{-1} = m^2 + \Sigma J_{-1-i} h_{-i}.$$

Hence the procedure is (a) calculate the h_i , (b) calculate the J_i , (c) form the products $J_p h_q$, (d) combine the products (c) to give the M_i and N_i . These quantities were quickly and conveniently obtained in this manner to the order required for obtaining c_0 to the order m^{11} .

In the solution of these equations the expressions

$$(2i+1+m+c)^2 + M_0$$

and

$$(2i+1+m-c)^2 + M_0 \text{ constantly recur.}$$

* "Researches in the Lunar Theory," *Collected Works*, vol. i. p. 317.

We denoted these d_i and D_i respectively, and, as is always possible, computed them one stage ahead of the stage in which they were required. The plan of solution consists in writing out the equations to the full extent required for the order intended for c_0 , one equation resulting from equating to zero the coefficient of each power of ζ .

We put

$$\begin{aligned}\epsilon_0 &= \frac{1}{4} + y, \\ \epsilon'_0 &= -\frac{3}{4} + y,\end{aligned}$$

forms suggested at once by Brown's numerical results and by his assumption that $\epsilon_0 - \epsilon'_0 = 1$. One of these quantities is of course at our disposal. We can now write down the equations in a form fit for solution. I write three of the eighteen necessary to proceed as far as m^8 to show their nature.

From Coefficient of	ζ^c	$d_0(\frac{1}{4} + y) + \epsilon_1 M_1 + \epsilon_{-1} M_{-1} + \epsilon_2 M_2 + \epsilon_{-2} M_{-2}$ $+ N_0(-\frac{3}{4} + y) + \epsilon'_1 N_1 + \epsilon'_{-1} N_{-1} + \epsilon'_2 N_2 + \epsilon'_{-2} N_{-2} = 0.$
	ζ^{-2+c}	$d_{-1}\epsilon_{-1} + M_1(\frac{1}{4} + y) + M_1\epsilon_{-2} + M_2\epsilon_1$ $+ N_0\epsilon'_1 + N_{-1}(-\frac{3}{4} + y) + N_1\epsilon'_2 + N_{-2}\epsilon'_{-1} = 0.$
	ζ^{6-c}	$D_3\epsilon'_{-3} + M_3(-\frac{3}{4} + y) + M_2\epsilon'_1 + M_1\epsilon'_2$ $+ N_0\epsilon_{-3} + N_1\epsilon_{-2} + N_2\epsilon_{-1} + N_3(\frac{1}{4} + y) = 0.$

We adopted a very convenient notation for this purpose. The symbol (hkl) was used to denote the coefficient of m^l in the quantity h with suffix k . If the suffix is negative, 10 is added. Thus we put

$h = 1$ for c_0	5 for d	$\left. \begin{array}{l} \text{(Terms in } c_0 \text{ and } y \\ \text{have, of course, only} \\ \text{two-letter symbols.)} \end{array} \right\}$
2 „ y	6 „ D	
3 „ ϵ	8 „ N	
4 „ ϵ'	9 „ M	

and as an example (318) stands for the coefficient of m^8 in ϵ_1 .

We write at once

$$c_0 = 1 + m - \frac{3}{2^2}m^2 + (13)m^3 + (14)m^4 + (15)m^5 + \dots$$

$$y = (22)m^2 + (23)m^3 + (24)m^4 + \dots$$

The solution of the equations now becomes a mere matter of routine. Each is solved in turn to the highest power of m possible, having regard to the known quantities involved in the equation. Having completed a whole round of the equations, we start again, and it will be found that new known quantities are at our disposal sufficient to enable us to obtain the coefficients of a new power of

m by performing another round. Let us write out, for example, the equation derived from the coefficient of $\zeta^c m^8$. Written out, this is:—

$$\begin{aligned} \frac{1}{4}d_0 + d_0y + \epsilon_1 M_1 + \epsilon_{-1} M_1 + \epsilon_2 M_2 + \epsilon_{-2} M_2 \\ - \frac{3}{4}N_0 + N_0y + \epsilon'_1 N_1 + \epsilon'_{-1} N_{-1} + \epsilon'_2 N_2 + \epsilon'_{-2} N_{-2} = 0. \end{aligned}$$

From the values obtained and tabulated in previous rounds each separate term is evaluated thus:—

$\frac{1}{4}d_0$	$\frac{1}{4}(508) = (18) - 859\ 592\ 077 \div 2^{17}3^3$
d_0y	$(506)(22) + (505)(23) + (504)(24) + (503)(25) + (502)(26) \\ + (501)(27) + 500\ 28 = \frac{9}{2}(28) + 405\ 808\ 383 \div 2^{21}$
$\epsilon_1 M_1$	$(312)(916) + (313)(915) + (314)(914) + (315)(913) \\ + (316)(912) = 1\ 143\ 542 \div 2^{15}3^2$
$\epsilon_{-1} M_1$	$(391)(917) + (392)(916) + (393)(915) + (394)(914) \\ + (395)(913) + (396)(912) = -1\ 296\ 939\ 119 \div 2^{19}3^2$
$\epsilon_2 M_2$	$(324)(924) = 5841 \div 2^{15}$
$\epsilon_{-2} M_2$	$(383)(925) = -21\ 549 \div 2^{15}$
$-\frac{3}{4}N_0$	$-\frac{3}{4}(808) = 139\ 638\ 557 \div 2^{19}3^2$
N_0y	$(806)(22) + (805)(23) + (804)(24) + (803)(25) + (802)(26) \\ + (801)(27) + (800)(28) = \frac{3}{2}(28) + 1\ 406\ 659\ 011 \div 2^{21} \cdot 3$
$\epsilon'_1 N_1$	$(411)(817) + (412)(816) + (413)(815) + (414)(814) \\ + (415)(813) + (416)(812) = 2\ 122\ 089\ 117 \div 2^{19} \cdot 3^3$
$\epsilon'_{-1} N_{-1}$	$(492)(896) + (493)(895) + (494)(894) + (495)(893) \\ + (496)(892) = -16\ 892\ 363 \div 2^{18}3^3$
$\epsilon'_2 N_2$	$(423)(825) + (424)(824) = 1\ 094\ 178 \div 2^{15}$
$\epsilon'_{-2} N_{-2}$	$(484)(884) = 615 \div 2^{18}$

and adding we get an equation for $(18) + 6(28)$ with a known right-hand side. In the same round we obtain a similar equation from the coefficient of $\zeta^{-c} m^8$ giving the value of (28) . Hence we obtain the value of (18) , *i.e.* the coefficient of m^8 in c_0 .

A complication arises from the terms in ζ^{-2+c} and ζ^{2-c} which need not be elaborated. It causes a postponement of the solution for the coefficients ϵ_{-1} and ϵ'_1 which correspond to those indices. A precisely analogous phenomenon occurs in previous lunar theories, and here it presents no especial analytical difficulty. In this kind of algebraical work the labour becomes so heavy that to ensure any certainty of accuracy some independent check must be devised. Andoyer (*loc. cit.*) actually performed his solutions by two totally distinct processes, one with polar and the other with rectangular co-ordinates. Dr. Cowell and I throughout worked on the plan that, while one of us obtained the results by solving the equations, the other checked them by the wholly different process of substituting the results into the equations, thus making the latter identities.

The further test of accuracy was also applied to the coefficient of m^{10} in c_0 of comparing it with Hill's coefficient, after allowing for the errors in the a_i , which of course vitiated his work. When this is done Hill's coefficient is identical with our own. The same process applied to Hill's coefficient of m^{11} , however, though it improves its accuracy, does not bring it into agreement with that printed below. Some further computing error must have crept into his work, a result not at all surprising to anyone who will take the trouble to follow the extraordinary mass of algebra through which he was bound to travel. The coefficient of m^{10} now published is certainly accurate, and I can say with great confidence that, although it lacks the final confirmation of agreement with Hill's value when corrected for the errors in the a_i , the coefficient of m^{11} is correct also.

I may add that as a result of this work Dr. Cowell and I are both convinced that any attempt to obtain the next term in m^{12} would involve so heavy a labour and one so disproportionate to any value attaching to it as to put its computation quite out of the question, at any rate until some better method is discovered.

The results for the ϵ_i , ϵ'_i , a_i , M_i , N_i , and for c_0 are given below in tabular form.

Hill's erroneous values of the coefficients of m^{10} and m^{11} are given by him as

$$-\frac{1\ 547\ 804\ 933\ 375\ 567}{2^{25}\ 3^5\ 5} m^{10} - \frac{818\ 293\ 211\ 836\ 767\ 367}{2^{28}\ 3^6\ 5^2} m^{11}.$$

There is a distinct regularity in the diminishing ratio of the terms in c_0 . Reduced to numbers and roughly extrapolated to infinity, I find, rather to my surprise, that the series is in close agreement with its true numerical value as calculated by Hill.*

I need hardly say that the errors in Hill's work now pointed out do not affect the numerical lunar theory of to-day. In the first place, Professor Brown has obtained all his results by independent computation. But beyond this, I have myself repeated the whole of the numerical work contained in Hill's two classical papers to fifteen significant figures, including the values of the a_i and c_0 . I find the results as printed are quite accurate. It is, however, to be observed that, contrary to the impression conveyed by the perusal of these papers, in order to obtain the results it is frequently necessary to include terms of the fourteenth and even sixteenth order before the end figures can be pronounced accurate, and Dr. Hill must have done this, since his end figures are practically without exception correct. There is one error of three or four units in one of the functions, which he denotes U_i , but I find it does not affect the final value of c_0 by so much as a unit in the fifteenth place of decimals.

* "Motion of the Perigee," *Collected Works*, vol. i. p. 270.

Series a , giving Coefficients of ξ^3 in u_0 . $a_0 = 1$.Power
of m .

a_1	a_2	a_3	a_4
m^0 ...	...	...	...
m^1 ...	...	...	...
m^2 $\frac{3}{2^4}$	...	...	...
m^3 $\frac{1}{2}$	...	...	...
m^4 $\frac{7}{2^2 \cdot 3}$	$\frac{25}{2^8}$	...	...
m^5 $\frac{11}{2^2 \cdot 3^2}$	$\frac{803}{2^7 \cdot 3 \cdot 5}$	...	...
m^6 $-\frac{30749}{2^{12} \cdot 3^3}$	$\frac{6109}{2^5 \cdot 3^2 \cdot 5^2}$	$\frac{833}{2^{13} \cdot 3}$	...
m^7 $-\frac{1010521}{2^{11} \cdot 3^4 \cdot 5}$	$\frac{897599}{2^8 \cdot 3^3 \cdot 5^3}$	$\frac{27943}{2^{11} \cdot 5 \cdot 7}$	...
m^8 $-\frac{18445871}{2^{10} \cdot 3^5 \cdot 5^2}$	$\frac{237203647}{2^{16} \cdot 3^2 \cdot 5^4}$	$\frac{12275527}{2^{10} \cdot 3^2 \cdot 5^2 \cdot 7^2}$	$\frac{3537}{2^{16}}$
m^9 $-\frac{2114557853}{2^{12} \cdot 3^6 \cdot 5^3}$	...	...	...
m^{10} $-\frac{5617623210853}{2^{19} \cdot 3^7 \cdot 5^4}$	...	...	...
m^{11} ...	...	...	...

Series a , giving Coefficients of ξ^{2i} in u_0 .

Power of m .	a_{-1} .	a_{-2} .	a_{-3} .	a_{-4} .
m^0	...	...	...	...
m^1	...	...	...	...
m^2	$\frac{19}{2^4}$	...	...	...
m^3	$-\frac{5}{3}$	...	...	...
m^4	$-\frac{43}{2^2 \cdot 3}$	...	...	...
m^5	$-\frac{14}{3^3}$	$\frac{23}{2^7 \cdot 5}$	...	...
m^6	$-\frac{7381}{2^{10} \cdot 3^4}$	$\frac{299}{2^5 \cdot 3 \cdot 5^2}$	$\frac{1}{2^6 \cdot 3}$	...
m^7	$\frac{3574153}{2^{11} \cdot 3^5 \cdot 5}$	$\frac{56339}{2^8 \cdot 3^2 \cdot 5^3}$	$\frac{7477}{2^{11} \cdot 3 \cdot 5 \cdot 7}$	...
m^8	$\frac{55218889}{2^9 \cdot 3^6 \cdot 5^2}$	$\frac{238200053}{2^{16} \cdot 3^3 \cdot 5^4}$	$\frac{65239}{2^9 \cdot 5^2 \cdot 7^2}$	$\frac{23}{2^{11} \cdot 3}$
m^9	$\frac{13620153029}{2^{12} \cdot 3^7 \cdot 5^3}$	...	...	...
m^{10}	$\frac{32912081196529}{2^{19} \cdot 3^8 \cdot 5^4}$	...	...	...
m^{11}	...	...	...	...

Series M.

Power of m .	M_0 .	M_1 .	M_2 .	M_3 .	M_4 .
0	$\frac{1}{2}$	...	...	...	...
1	1	...	...	...	...
2	$\frac{5}{2^2}$	$\frac{3}{2^2}$	...	...	...
3	0	$\frac{19}{2^3}$	...	...	...
4	$-\frac{9}{2^6}$	$\frac{10}{3}$	$\frac{33}{2^5}$	...	...
5	2	$\frac{43}{2 \cdot 3^2}$	$\frac{2937}{2^7 \cdot 5}$	...	...
6	$\frac{17}{3}$	$\frac{18709}{2^{10} \cdot 3^3}$	$\frac{23051}{2^5 \cdot 3 \cdot 5^2}$	$\frac{1393}{2^{10}}$	...
7	$\frac{15}{2}$	$\frac{759413}{2^{11} \cdot 3^4 \cdot 5}$	$\frac{97051}{2^6 \cdot 5^3}$	$\frac{562041}{2^{11} \cdot 5 \cdot 7}$	...
8	$\frac{2701801}{2^{14} \cdot 3^3}$	$\frac{6675059}{2^9 \cdot 3^5 \cdot 5^2}$	$\frac{167206573}{2^{10} \cdot 3^3 \cdot 5^4}$	$\frac{40986497}{2^9 \cdot 3 \cdot 5^2 \cdot 7^2}$	$\frac{7177}{2^{12}}$
9	$\frac{122957}{2^6 \cdot 3^4 \cdot 5}$	$-\frac{41991161}{2^8 \cdot 3^6 \cdot 5^3}$	...	...	...
10	$\frac{1260881}{2^{10} \cdot 3^4 \cdot 5}$	$-\frac{4528082998913}{2^{18} \cdot 3^7 \cdot 5^4}$	...	...	...
11	$-\frac{291394307}{2^8 \cdot 3^6 \cdot 5^3}$	...	...	...	...

Series N.
 N_0 to N_4 .

Power of m.	N_0 .	N_1 .	N_2 .	N_3 .	N_4 .
m^0	$\frac{3}{2}$	...	...	...	...
m^1	3	...	...	...	...
m^2	$\frac{9}{2^2}$	$\frac{69}{2^4}$	...	...	...
m^3	0	$\frac{29}{2}$	...	...	...
m^4	$-\frac{417}{2^7}$	$\frac{2137}{2^6 \cdot 3}$	$\frac{4497}{2^9}$	...	...
m^5	$-\frac{551}{2^6}$	$\frac{335}{2 \cdot 3^2}$	$\frac{53121}{2^8 \cdot 5}$	...	...
m^6	$-\frac{4993}{2^8}$	$-\frac{5737}{2^6 \cdot 3^3}$	$\frac{7201393}{2^{10} \cdot 3 \cdot 5^2}$	$\frac{31549}{2^{11}}$	...
m^7	$-\frac{623}{2 \cdot 3^2}$	$-\frac{1938205}{2^9 \cdot 3^4}$	$\frac{3135247}{2^6 \cdot 3 \cdot 5^3}$	$\frac{1680433}{2^9 \cdot 5 \cdot 7}$	...
m^8	$\frac{139638557}{2^{17} \cdot 3^3}$	$-\frac{37124881}{2^{10} \cdot 3^5}$	$\frac{56445087089}{2^{16} \cdot 3^3 \cdot 5^4}$	$\frac{4183715257}{2^{12} \cdot 3 \cdot 5^2 \cdot 7^2}$	$\frac{3253535}{2^{17}}$
m^9	$-\frac{3068447}{2^{16} \cdot 5}$	$-\frac{613012001}{2^9 \cdot 3^6 \cdot 5}$	...	...	...
m^{10}	$\frac{124884596449}{2^{18} \cdot 3^5 \cdot 5^2}$	$-\frac{7436037981679}{2^{18} \cdot 3^7 \cdot 5^2}$	...	...	...
m^{11}	$\frac{11389298779}{2^9 \cdot 3^6 \cdot 5^3}$	...	...	...	...

Series N.

 N_{-1} to N_{-4} Power
of m . N_{-1} N_{-2} N_{-3} N_{-4}

m^0	...	...	...	...
m^1	...	...	...	...
m^2	$\frac{27}{2^4}$	...	...	...
m^3	$-\frac{1}{2^2}$	...	...	...
m^4	$-\frac{217}{2^6 \cdot 3}$	$\frac{123}{2^9}$	...	...
m^5	$-\frac{77}{2 \cdot 3^2}$	$\frac{823}{2^6 \cdot 5}$	...	...
m^6	$-\frac{143911}{2^{11} \cdot 3^3}$	$\frac{27899}{2^{10} \cdot 3 \cdot 5^2}$	$\frac{1}{2^3}$	...
m^7	$\frac{183157}{2^9 \cdot 3^4}$	$-\frac{52987}{2^6 \cdot 3^2 \cdot 5^3}$	$\frac{555}{2^9}$	...
m^8	$\frac{63951119}{2^{12} \cdot 3^6 \cdot 5}$	$-\frac{16228799}{2^{15} \cdot 5^4}$	$\frac{22213}{2^6 \cdot 5^2 \cdot 7}$	$\frac{135}{2^9}$
m^9	$\frac{175722601}{2^9 \cdot 3^6 \cdot 5^2}$	...	...	...
m^{10}	$\frac{476841356501}{2^{16} \cdot 3^7 \cdot 5^3}$	...	...	...
m^{11}	...	...	...	...

Series ϵ , giving Coefficients of ϵ^{n+c} in series for u_0 .

ϵ_0 to ϵ_4 .

Power of m .	ϵ_0 .	ϵ_1	ϵ_2 .	ϵ_3 .	ϵ_4 .
m^0	$\frac{1}{4}$	...	...	...	...
m^1	0	...	...	...	...
m^2	$\frac{3}{2^4}$	$\frac{3}{2^4}$	...	...	...
m^3	$\frac{651}{2^{10}}$	$\frac{27}{2^6}$	...	...	...
m^4	$\frac{2817}{2^{12}}$	$\frac{697}{2^8 \cdot 3}$	$\frac{177}{2^{10}}$	...	...
m^5	$\frac{139487}{2^{16}}$	$\frac{16649}{2^{12} \cdot 3^2}$	$\frac{3447}{2^{10} \cdot 5}$	...	...
m^6	$\frac{1154527}{2^{15} \cdot 3}$	$\frac{15499}{2^{13} \cdot 3^2}$	$\frac{387233}{2^{12} \cdot 3^5 \cdot 5^2}$	$\frac{529}{2^{10} \cdot 3}$	...
m^7	$\frac{1311946431}{2^{21} \cdot 3^2}$	$\frac{135369541}{2^{18} \cdot 3^4 \cdot 5}$	$\frac{29701193}{2^{16} \cdot 3^5 \cdot 5^2}$	$\frac{266307}{2^{13} \cdot 5 \cdot 7}$	...
m^8	$\frac{9536459185}{2^{20} \cdot 3^3}$	$\frac{3654513337}{2^{20} \cdot 3^5 \cdot 5}$	$\frac{92012820161}{2^{20} \cdot 3^3 \cdot 5^4 \cdot 7}$	$\frac{705800387}{2^{17} \cdot 3^2 \cdot 5 \cdot 7^2}$	$\frac{46825}{2^{18}}$
m^9	$\frac{3148434304651}{2^{28} \cdot 3^4}$	$\frac{1826839250773}{2^{22} \cdot 3^5 \cdot 5^3}$	...	...	...
m^{10}	$\frac{1079935871017417}{2^{28} \cdot 3^3 \cdot 5^2}$	...	...	...	...
m^{11}	$\frac{19306876744210342787}{2^{33} \cdot 3^6 \cdot 5^3}$	...	...	...	...

Series ϵ , giving Coefficients of ζ^{23+c} in u_e . ϵ_{-1} to ϵ_{-4} and c_0 .

Power of m .	ϵ_{-1} .	ϵ_{-2} .	ϵ_{-3} .	ϵ_{-4} .	c_0 .
m^0	...	...	...	...	1
m^1	$-\frac{45}{2^5}$	...	...	...	1
m^2	$-\frac{555}{2^7}$	...	...	...	$\frac{3}{2^2}$
m^3	$-\frac{22441}{2^{11}}$	$-\frac{15}{2^8}$	...	...	$-\frac{201}{2^5}$
m^4	$-\frac{63583}{2^{10} \cdot 3}$	$-\frac{193}{2^9}$	...	...	$-\frac{2367}{2^7}$
m^5	$-\frac{22740977}{2^{16} \cdot 3^2}$	$-\frac{114707}{2^{14} \cdot 5}$	$\frac{75}{2^{12}}$	...	$-\frac{111749}{2^{11}}$
m^6	$-\frac{24742523}{2^{13} \cdot 3^3}$	$-\frac{15591029}{2^{15} \cdot 3^2 \cdot 5^2}$	$\frac{7469}{2^{14} \cdot 3}$	...	$-\frac{4095991}{2^{13} \cdot 3}$
m^7	$-\frac{383711325709}{2^{23} \cdot 3^4}$	$\frac{15704738987}{2^{19} \cdot 3^3 \cdot 5^3}$	$\frac{17805751}{2^{18} \cdot 3 \cdot 5 \cdot 7}$	$\frac{1305}{2^{16}}$	$-\frac{332582087}{2^{16} \cdot 3^2}$
m^8	$-\frac{31516547013223}{2^{23} \cdot 3^5 \cdot 5}$	$\frac{1048109872977}{2^{18} \cdot 3^4 \cdot 5^4}$	$\frac{11635634287}{2^{19} \cdot 3^2 \cdot 5^2 \cdot 7^2}$	$\frac{330123}{2^{18} \cdot 7}$	$-\frac{15108211789}{2^{18} \cdot 3^3}$
m^9	$-\frac{76750587110974937}{2^{28} \cdot 3^6 \cdot 5^2}$	...	...	...	$-\frac{5975332916861}{2^{23} \cdot 3^4}$
m^{10}	...	...	...	...	$-\frac{1547775442175567}{2^{25} \cdot 3^5 \cdot 5}$
m^{11}	...	...	...	...	$-\frac{818429336556024967}{2^{28} \cdot 3^6 \cdot 5^2}$

Series ϵ' , giving Coefficients of ζ^{2k-c} in u_c . ϵ'_0 to ϵ'_4 .Power
of m .

ϵ'_0	ϵ'_1	ϵ'_2	ϵ'_8	ϵ'_4
m^0 $-\frac{3}{4}$	...	...	...	...
m^1 0	$\frac{15}{2^5}$	...	...	...
m^2 $\frac{3}{2^4}$	$\frac{277}{2^7}$	...	...	...
m^3 $\frac{651}{2^{10}}$	$\frac{36329}{2^{11} \cdot 3}$	$\frac{45}{2^7}$	...	...
m^4 $\frac{2817}{2^{12}}$	$\frac{87599}{2^{10} \cdot 3^2}$	$\frac{137}{2^6}$	...	...
m^5 $\frac{139487}{2^{16}}$	$\frac{2639689}{2^{16} \cdot 3^3}$	$\frac{908809}{2^{13} \cdot 3 \cdot 5}$	$\frac{2655}{2^{13}}$	...
m^6 $\frac{1154527}{2^{15} \cdot 3}$	$-\frac{104415137}{2^{15} \cdot 3^4}$	$\frac{129979907}{2^{10} \cdot 3^2 \cdot 5^2}$	$\frac{79593}{2^{15}}$	...
m^7 $\frac{1311946431}{2^{21} \cdot 3^2}$	$-\frac{842107318607}{2^{23} \cdot 3^5 \cdot 5}$	$\frac{12526925651}{2^{17} \cdot 3^3 \cdot 5^3}$	$\frac{36573685}{2^{19} \cdot 7}$	$\frac{21160}{2^{16}}$
m^8 $\frac{9536459185}{2^{20} \cdot 3^3}$	$\frac{49757640175171}{2^{23} \cdot 3^6 \cdot 5^2}$	$\frac{419081003537}{2^{20} \cdot 3^3 \cdot 5^4}$	$\frac{18346696987}{2^{19} \cdot 5^2 \cdot 7^2}$	$\frac{3945485}{2^{16} \cdot 3 \cdot 7}$
m^9 $\frac{31484343004651}{2^{28} \cdot 3^4}$	$\frac{273856047367759949}{2^{28} \cdot 3^7 \cdot 5^3}$	...	...	...
m^{10} $\frac{1079935871017417}{2^{28} \cdot 3^3 \cdot 5^2}$	...	...	...	...
m^{11} ...	...	...	...	...

Series ϵ' , giving Coefficients of ϵ'^{2i-c} in u_e .

ϵ'_{-1} to ϵ'_{-4} .

Power of m .

ϵ'_{-1}	ϵ'_{-2}	ϵ'_{-3}	ϵ'_{-4}
...	...	...	...
...	...	...	...
$-\frac{1}{2^5}$	...	...	...
$\frac{29}{2^6 \cdot 3}$	...	...	...
$\frac{35}{2^9 \cdot 3^2}$	$\frac{5}{2^9}$	...	...
$-\frac{33779}{2^{11} \cdot 3^3}$	$\frac{967}{2^{10} \cdot 3 \cdot 5}$	...	...
$-\frac{1539359}{2^{14} \cdot 3^4}$	$\frac{23029}{2^{12} \cdot 3^2 \cdot 5}$	$\frac{87}{2^{13}}$	...
$-\frac{170697379}{2^{15} \cdot 3^5 \cdot 5}$	$\frac{38279881}{2^{17} \cdot 3^3 \cdot 5^3}$	$\frac{7201}{2^{14} \cdot 7}$	...
$-\frac{14868011963}{2^{20} \cdot 3^6}$	$-\frac{1441200427}{2^{19} \cdot 3^2 \cdot 5^4 \cdot 7}$	$\frac{39780541}{2^{16} \cdot 3 \cdot 5^2 \cdot 7^2}$	$\frac{3815}{2^{17} \cdot 3}$
$-\frac{192109063495339}{2^{23} \cdot 3^7 \cdot 5^3}$	...	...	...
...	...	...	...
...	...	...	...

m^{11}

The Motion in Longitude of the Red Spot on Jupiter. By
Rev. T. E. R. Phillips, M.A.

The present zero meridian of System II., which is used in the reduction of observations of extra-equatorial spots on Jupiter, was adopted by Marth in his ephemeris for 1892-3 (*M.N.*, 52, 524). Its daily rate— $87^{\circ}27'$, corresponding to a rotation period of $9^{\text{h}} 55^{\text{m}} 40^{\text{s}}.632$ —had been in use since 1886, but the position of the zero meridian had been repeatedly changed, the meridian of the 1892-3 ephemeris being a return to that adopted in 1890. From 1892 to 1914 (inclusive) it remained in use without further interruption; but in the 1915 ephemeris, which was communicated to the British *Nautical Almanac* by Washington, the zero meridian was changed with a view to bringing it into agreement with the position of the Red Spot. This altered meridian, however, does not seem to have been employed, at any rate in England, observers preferring to apply the correction needed to make the meridian of reference continuous with that of previous years. In the 1916 ephemeris the old zero meridian was restored. It may be worth while to draw attention to this, as the *Nautical Almanac* for that year (p. 599) merely states that "System II. is not continuous with 1915."

It follows from the above remarks that the published longitudes of extra-equatorial spots (except possibly a few relating to the 1915 apparition) are mutually comparable, and this fixed meridian affords a valuable means of recording the drift in longitude of markings on the planet's surface from year to year.

The main purpose of this note is to draw attention to the fact that the great Red Spot has performed during the past seventeen years a complete revolution round the globe with reference to this adopted meridian.

At the opposition of 1894 Dec. 22 the zero meridian of System II. passed through, or very close to, the centre of the Red Spot; but, the rotation period of the latter being slightly the greater, the spot drifted steadily in the direction of increasing longitude. By the middle of 1901 July it followed the zero meridian by about 78 minutes, so that its longitude was 47° , but an acceleration then set in, and the swifter motion has continued with but little interruption, though irregularly, to the present day.

In 1902, 1904, 1906, 1908, 1910, 1913-14, and 1917-18 conjunctions have occurred with the S. Tropical Disturbance, which is situated in the same latitude, but moves at a swifter rate. This object has shown extraordinary developments since it was first observed by Molesworth as a protuberance from the S. edge of the S. Equatorial belt on 1901 Feb. 28, and during the apparition recently closed it extended over 170° of longitude! The motion is now much slower than formerly, and, since that of the Red Spot has increased, the intervals between mid-conjunction have widened, while the time covered by such conjunctions has become much extended.

Transit observations made at successive apparitions show that in each case conjunction has been accompanied by a very definite acceleration of the Red Spot. This has been especially marked during the last few months; in fact, from the beginning of January to the close of the observations the object exhibited the abnormally short rotation period of $9^h 55^m 31^s.5$! The mean period derived from the whole series of observations made at Headley, and extending over nine months, was $9^h 55^m 33^s.1$.

No observations of the Red Spot were obtained in April, but an opportunity presented itself on the evening of May 12. At the actual moment of the Spot's central meridian transit the planet was obscured by passing cloud, but from observations made a few minutes before and after, together with an observation of the "shoulder," the time of passage was estimated to be $7^h 30^m$ G.M.T., making the longitude $45^\circ.0$. The Spot has therefore completed a journey round the planet since it began to retrograde in longitude in the summer of 1901.

The following table gives the longitude (System II.) of the Red Spot, or the hollow in which it lies, at successive oppositions; the rotation period deduced from transit observations during the planet's apparition; and the rotation period during the intervals from one opposition to the next. The figures in the last column but one are deduced from the observations of various observers.

TABLE I.

Date of Opposition.		Longitude (System II.).	Rotation Period.					
			During Apparition.			Between Oppositions.		
Year.	Day.		h	m	s	h	m	s
1894	Dec. 22.6	$0^\circ \pm$	...			9	55	41.4
1896	Jan. 24.1	7	...			"	"	41.5
1897	Feb. 23.1	15	...			"	"	41.4
1898	Mar. 25.5	22	9	55	42.1	"	"	41.6
1899	Apr. 25.5	31	"	"	42.0	"	"	41.6
1900	May 27.3	40	"	"	41.8	"	"	41.2
1901	June 30.9	46	"	"	40.8	"	"	40.2
1902	Aug. 5.2	42	"	"	39.3	"	"	39.8
1903	Sept. 11.7	34	"	"	41.3	"	"	39.7
1904	Oct. 18.5	25	"	"	40.2	"	"	40.9
1905	Nov. 23.9	28	"	"	41.2	"	"	39.6
1906	Dec. 28.1	18	"	"	41.5	"	"	41.1
1908	Jan. 29.4	23	"	"	40.8	"	"	40.0
1909	Feb. 28.3	17	"	"	41.9	"	"	39.7
1910	Mar. 30.7	8	"	"	38.4	"	"	36.5
1911	Apr. 30.7	328	"	"	38.3	"	"	38.2
1912	May 31.9	304	"	"	38.8	"	"	36.2
1913	July 5.1	261	"	"	34.7	"	"	34.7
1914	Aug. 10.4	203	"	"	36.5	"	"	37.4
1915	Sept. 17.0	171	"	"	37.5	"	"	36.4
1916	Oct. 23.6	130	"	"	36.7	"	"	35.2
1917	Nov. 28.7	77	"	"	33.1	"	"	

There are three points of interest arising out of the above figures:—

1. It is evident that any attempt to fix a zero meridian which shall continue for any length of time to coincide with the Red Spot is doomed to failure in consequence of the latter's wanderings in longitude.

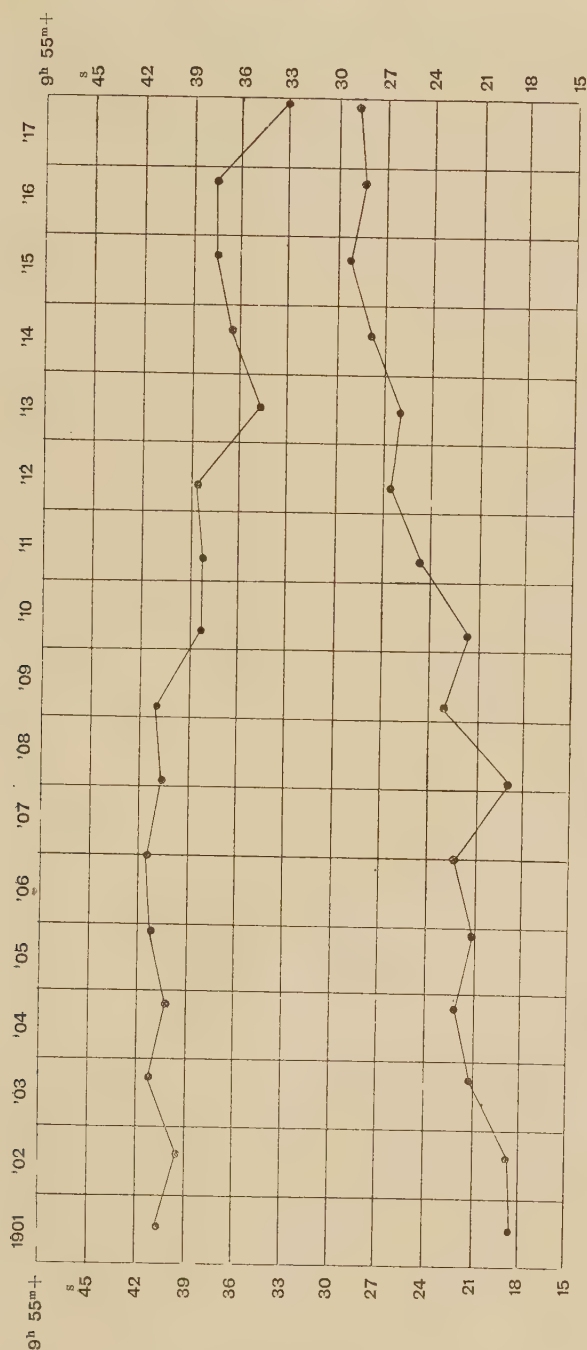
2. Many, though not quite all, of the irregularities of motion are associated with the conjunctions of the Spot and S. Tropical Disturbance. As already remarked, whenever the Disturbance overtakes the Red Spot the motion of the latter is accelerated. It seems as if it were swept along by the more swiftly moving stream, as M. Antoniadi pointed out some years ago. But there is one circumstance which it is somewhat difficult to reconcile with this theory—we should expect the Disturbance in the act of accelerating the Red Spot to be, if anything, itself retarded, whereas the opposite appears to be the case. Table II. gives the mean rotation periods of the Disturbance (taken as a whole) deduced from the work of various observers at successive apparitions, and when these are plotted with the figures of column 3 in Table I., it is plainly seen that—neglecting small discordances probably arising from observational or kindred errors—the Disturbance shows changes of rate which are really *in sympathy* with those of the Red Spot instead of exhibiting any effect of reaction. (See, too, *B.A.A. Journal*, 26, 12, where reference is made to the agreement in this respect between the Red Spot and S. Tropical Spots generally.)

TABLE II.

Mean Rotation Period of S. Tropical Disturbance.

Apparition.	h	m	s	Apparition.	h	m	s
1901	9	55	18.5	1909-10	9	55	21.6
1902	„	„	18.8	1911	„	„	24.7
1903	„	„	21.1	1912	„	„	26.6
1904	„	„	22.1	1913 *	„	„	26.0
1905-6	„	„	21.2	1914	„	„	27.9
1906-7	„	„	22.3	1915	„	„	29.3
1907-8	„	„	19.0	1916-17	„	„	28.2
1908-9	„	„	23.1	1917-18	„	„	28.9

It is clear from the diagram that the rotation periods of the two objects are gradually approaching one another; but instead of maxima in the one case coinciding with minima in the other, the oscillations which are superimposed on the main curves are in similar directions. The natural suggestion is that we are here concerned not with one object acting on another, but rather with some deep-seated cause affecting the Red Spot and S. Tropical Disturbance simultaneously. But if so, it remains to be explained why these changes of period are practically only in evidence at times of conjunction!



Rotation Period of Red Spot (upper) and South Tropical Disturbance (lower).

3. The third point of interest arises from a comparison of columns 3 and 4 in Table I., especially when these are considered in connection with the interesting paper by Mr. F. Sargent "On the Rotation of the S. Tropical Disturbance" in *M.N.*, 77, 68. In that paper Mr. Sargent points out that the periods derived from observations made during successive apparitions are very sensibly longer than those derived from a comparison of observations at the close of an apparition with those made at the beginning of the next. He further states that his observations of the Red Spot Hollow and another object show no such effect, and concludes that the phenomenon is therefore real, and not due to any form of error.

But the effect to which Mr. Sargent drew attention is not really new, nor is it the case that it is not apparent in observations of the Red Spot. In the years to which Mr. Sargent refers it happened to be masked by the greater inequalities exhibited in the motion of the Spot, but it comes out quite clearly when the whole series of observations is considered. Turning to Table I, the mean value of column 4 from 1898 to 1916 inclusive is $9^h 55^m 39^s.6$, whereas the mean of the last column, beginning with the figures for the interval between 1897 and 1898, is $9^h 55^m 39^s.0$ —a difference of $0^s.6$, which is quite perceptible.

Moreover, Mr. Sargent will find the matter discussed so long ago as 1905 in *Observatory*, 28, 344, 389, etc., and I think that Mr. A. S. Williams's suggested explanation is a sound one, viz. that we have here the effect of *phase-darkening*, which causes a Spot's transit to be recorded too early at the beginning of an apparition, and too late at the close. The varying slant of the belts before and after opposition may also help to cause such an effect.

The differences shown by Mr. Sargent are much larger than those I have found in the figures relating to the Red Spot, but his plan of determining the rotation period during the time of the planet's invisibility, rather than from opposition to opposition, tends naturally to *exaggerate* the phenomenon, assuming the above explanation to be correct, by basing the period on observations which are too late and too early respectively, instead of on observations which, being made at opposition, are obviously free from any such sources of error.

If my contention is sound, it follows that rotation periods derived from a series of observations during the planet's visibility are all slightly too long, and that the most reliable results are those which are based on a comparison of the longitudes at successive oppositions. Difficulties of identification, however, render this method impracticable in all but a very few instances.

An Example of the Determination of a Minute Periodic Variation, as Illustrative of the Law of Errors. By S. Chapman, M.A., D.Sc.

In a paper recently communicated to the Royal Meteorological Society, I have described the method and results of an attempt to determine the lunar semidiurnal variation of barometric pressure at Greenwich. Such a variation is to be expected as a consequence of tidal action upon the atmospheric "ocean" which envelops the earth, but its magnitude had not hitherto been ascertained, either theoretically or observationally, for any extra-tropical station. The results of the new investigation I regard as establishing the existence of a lunar tidal barometric variation at Greenwich of approximate amount:—

$$0.0091 \sin(2t + 114^\circ) \text{ in mm. of mercury,}$$

or

$$0.00036 \sin(2t + 114^\circ) \text{ in inches of mercury,}$$

where t ranges from 0° to 360° during the interval between any two successive upper transits of the moon. The considerations which support this conclusion, and the meteorological bearing of the result, are discussed in the paper cited. In the present note I wish to draw attention to the somewhat remarkable nature of the determination from the statistical standpoint.

The material used consisted of 6457 sequences of 25 hourly measures from the Greenwich photographic records of barometric pressure. These sequences each consisted of the measure for 23^h civil time on one day and the 24 successive hourly measures (0^h to 23^h) on the following day. These were regarded as representing the course of the barometer during one lunar day of duration 25 solar hours; this is nearly, though not quite, correct, the lunar day ranging from about $24^h 35^m$ to $25^h 5^m$, with a mean value of $24^h 50^m.5$. The 6457 sequences were drawn from the 64-year period 1854–1917, the days selected being those for which the range of barometric height did not exceed 0.1 inch.

The 25 hourly values in each sequence were retabulated on "lunar" sheets. On these the first value entered in any row was that for the solar hour at or next after lunar upper transit. The succeeding members in the sequence, to 23^h of the barometrically "quiet" day, then followed. The next entry was that for 23^h of the previous day, and was followed by the consecutive members of the sequence for the part of the quiet day before lunar transit. The sequence was thus broken into two parts, that preceding lunar transit being transposed, on the lunar sheets, to follow after that succeeding this epoch. The difference of lunar time between the measures on either side of the discontinuity in the lunar rows, *i.e.* the difference between the lunar times corresponding to 23^h on one solar day and the preceding, is $50^m.5$ on the average. The adopted method assumes it to be one solar hour, but the slight

error occurs at all lunar hours in the course of many rows. The mean lunar time corresponding to entries in the first lunar column is $0\frac{1}{2}$ solar hour, and thereafter the mean time interval between successive columns is one solar hour, or rather a little less, practically $\frac{1}{25}$ of the lunar day.

The measures on the original sheets were given to 0.001 inch of mercury: the scale of the photographic registers being fourfold, this represented approximately 0.1 mm. linear measure on the sheets. In retabulating the measures, however, the last figure, indicating the thousandths of an inch, was omitted, the previous figure being raised by one unit when the last figure exceeded 5. When the last figure was 5, the next *even* figure was adopted for the second decimal: thus, 29.875 would be read as 29.88, and 29.865 as 29.86. For convenience, also, a whole number of inches and tenths of an inch were omitted from all the entries in any one lunar row.

The solar diurnal barometric variation was not at first abstracted from the "lunar" entries, as it was expected that, in a large number of days, the distribution of solar times of lunar transits would be so uniform throughout the day that the solar variation would be eliminated by being averaged out. This actually occurred, very approximately, in regard to the whole set of material, but three different threefold subdivisions of the data were also made, in which it proves that the solar variation was not well eliminated. The subdivisions were made in order to test the accuracy of the results by their internal accordance: they consisted of (*a*) days from the three separate periods 1854-73, 1874-93, 1894-1917; and days drawn from the whole period 1854-1917 grouped, according to (*b*) season, *i.e.* summer (May to August), equinox (March, April, September, and October), and winter (November to February); and (*c*) lunar distance, these groups being referred to as the perigee, mean, and apogee groups.

Only for the first group (*a*) did the lunar semidiurnal terms in the mean lunar hourly inequalities derived from the lunar tabulations show at all reasonable accordance. It was therefore decided to correct the results by removing, to some extent, the solar diurnal variation. The annual mean solar variation determined from the twenty years 1854-73 was used for this purpose, no attention being paid to secular and seasonal changes in the variation. These least affect the semidiurnal component, so that this, the most important source of error in determining the lunar semidiurnal variation, was most effectively eliminated; with more trouble the lunar hourly inequalities could probably be improved further, but this was not considered necessary. The distribution of the solar times of lunar transit for the days in each of the nine subdivisions of the material was determined, and then, by a simple numerical process, the part of the hourly inequalities on the lunar sheets due to the presence of the mean solar hourly inequalities was calculated and allowed for. It was then found that the lunar semidiurnal terms from the nine sub-groups of data were brought into excellent accordance, considering the smallness of the quantities concerned.

The total and nine sub-groups of material afforded the ten lunar hourly inequalities, for the nominal solar hours $0\frac{1}{2}$, $1\frac{1}{2}$, . . . $24\frac{1}{2}$ after lunar transit, which are given in Table II. of the paper cited. They may also be readily inferred from the table here given. The semidiurnal component $0.0091 \sin (2t + 114^\circ)$, already mentioned, was deduced by harmonic analysis of the first inequality, relating to the whole period 1854-1917. The hourly values of this component are given in the second column of the table, and in the remaining columns are the differences between the ten observed inequalities and this computed inequality. The unit in this table is 0.001 mm. of mercury. At the foot of each column of discordances the numerical mean is written.

The remarkable feature of the result is that the whole amplitude of the variation, 0.00072 inch of mercury, is less than three-quarters of the last unit of measurement in the hourly values used, and this last unit, representing 0.1 mm. linear measure on the photographic sheets, is only just conveniently measurable with an ordinary scale. The graph of the mean inequality from the whole material, 1854-1917, is compared with the curve $0.0090 \sin (2t + 114^\circ)$, in a figure in the paper already referred to, and the mean discordance (*cf.* the table) is only 0.0026 mm. of mercury, or 0.00010 inch, one-tenth of the last unit of measurement. When originally considering the possibility of success before commencing on this investigation, I had estimated that this would be the amount of the accidental error to be expected in the mean lunar hourly inequalities. This estimate was based on the following rough calculation. The days selected for use were those on which the range of barometric height did not exceed 0.1 inch, or 100 units of one-thousandth of an inch. Taking the mean range as half this, or 50 units, I estimated the mean departure of any particular hourly value from the mean of the twenty-five as about 10 units. Regarding this as the accidental error of each value on the lunar sheets, the accidental error of the mean of any column, there being about 6400 rows, should be $10/\sqrt{6400}$ or $\frac{1}{8}$ unit, *i.e.* 0.00012 inch of mercury. This assumes that the well-known theorem concerning the error of the arithmetic mean of a large number of observations can be regarded as correct down to such small fractions of the last unit. This point is perhaps somewhat uncertain, but the present results shed an interesting light upon it.

Besides the mean discordance for the whole series of data, those for the nine subdivisions are given in the table. If these are divided by $\sqrt{(6457/\text{number of days in group})}$ they should theoretically become comparable with the mean discordance in the third column. The table shows how far this actually occurs. For the subdivision (a) the accordance is fairly close, and also, though less so, for the second group of three. In the third group this is not so, but it may be noticed that where the mean discordances are greatest the individual values partake least of the character of accidental errors, and it is possibly the case that this is due partly

to a residue of the diurnal component of the solar variation which has not been eliminated from the hourly inequalities.

The Mean Lunar Semidiurnal Barometric Variation at Greenwich, 1854-1917, and the Discordances, Relative to it, of the Observed Lunar Hourly Inequalities of Barometric Pressure.

The unit is 0.001 mm. of mercury.

Mean Solar Hours after Lunar Transit.	Hourly In- equalities of Mean Lunar Semidiurnal Barometric Variation.	Hourly Discordances.									
		Group of Data, and Number of Days in Group.									
		1854- 1917.	1854-73.	1874-93.	1894-17.	Summer.	Equinox.	Winter.	Perigee.	Mean.	Apogee.
		6457.	1867.	2079.	2511.	2817.	2052.	1578.	1999.	2733.	1725.
0½	7	-5	-8	-3	-6	1	-6	-16	-28	-5	19
1½	3	0	5	8	-10	2	-6	4	-22	-1	26
2½	-1	-1	0	6	-9	-2	8	-12	-24	3	18
3½	-5	-6	5	-2	-17	-11	1	-6	-19	-12	20
4½	-8	-4	1	3	-12	-12	1	5	-16	-10	21
5½	-9	-1	-5	4	-2	-13	11	5	-16	1	12
6½	-8	-3	-5	0	-3	-14	8	4	-16	-1	10
7½	-4	-2	3	-7	-1	-7	-5	12	-22	12	1
8½	0	-5	-1	-10	-4	-14	-7	13	-20	7	-8
9½	4	2	6	-5	4	-7	2	17	2	10	-10
10½	8	1	11	-11	3	-6	11	-3	9	1	-11
11½	9	1	10	-5	1	-5	13	-2	20	0	-17
12½	8	0	9	-9	2	-1	-2	6	28	-3	-27
13½	5	5	3	4	8	3	10	2	30	5	-23
14½	1	3	-4	3	9	7	-7	10	28	0	-21
15½	3	2	6	-9	8	4	-12	17	20	3	-20
16½	-7	-1	0	-8	2	-1	-10	9	21	-11	-13
17½	-9	5	10	-1	6	8	-1	8	19	4	-10
18½	-9	4	5	1	6	11	1	-2	20	-2	-5
19½	-6	4	3	3	5	11	-3	-2	13	-2	1
20½	-2	4	-6	12	5	22	-5	-16	12	1	0
21½	2	-4	-16	4	0	4	-3	-17	5	-5	-11
22½	6	0	-15	6	5	5	-2	-13	0	-2	3
23½	8	-1	-5	4	-1	4	2	-14	-20	2	17
24½	9	2	-11	11	3	11	-2	-9	-24	4	27
Mean numerical discordance.		2.6	6.1	5.6	5.3	7.4	5.6	9.0	18.2	4.3	14.0
Ditto. ÷ $\sqrt{\frac{6457}{\text{No. of days.}}}$		2.6	3.3	3.2	3.3	4.9	3.1	4.4	10.1	2.8	7.3

Note on the Pulsation Theory of Cepheid Variables.

By F. A. Lindemann.

It may be worth while to call attention to a method of testing the pulsation theory of Cepheid variables, and, if this theory is confirmed, of determining their parallaxes, even though the writer is at present not able to work out the problem in detail.

On the pulsation theory the variations in the radial velocity are due to the surface of the star approaching or receding. In this case the lines should not shift, strictly speaking, as there would be no change in radial velocity at the rim, but they should broaden on alternate sides. Assuming this to be true, and calling v the radial velocity corresponding to the centre of the disk, *i.e.* as deduced from the edge of the line, the velocity of the centre of gravity is obviously $\frac{1}{P} \int_0^P v dt$, where P is one whole period.

Further, the radius of the star at time t must be $R_0 + \int_0^t v dt - \frac{t}{P} \int_0^P v dt$, R_0 being the radius at zero time.

If T is the temperature at time t , the brightness of unit surface is $\frac{2\pi h}{c^2} \int_0^\infty \frac{\nu^3 \phi(\nu)}{e^{\frac{h\nu}{kT}} - 1} d\nu$, where $\phi(\nu)$ is the function connecting the

sensitiveness of the eye or the plate with the frequency ν . The apparent brightness of the star at time t is therefore given by

$$\frac{(R_0 + \int_0^t v dt - \frac{t}{P} \int_0^P v dt)^2}{d^2} \cdot \frac{2\pi h}{c^2} \int_0^\infty \frac{\nu^3 \phi(\nu)}{e^{\frac{h\nu}{kT}} - 1} d\nu.$$

The only unknown quantities in this expression are the radius R_0 and the distance of the star d , and these can obviously be easily determined by evaluating $\int v dt$ for two different times, and

$\int_0^\infty \frac{\nu^3 \phi(\nu)}{e^{\frac{h\nu}{kT}} - 1} d\nu$ for the temperatures corresponding to these times,

and equating the values so found to the observed magnitudes.

Professor Eddington was kind enough to point out that the maxima and minima of magnitude correspond closely to periods of maximum and minimum radial velocity, so that for any considerable change of temperature there has been only a small change in the radius. This fact, which would seem to indicate a slow flow of heat from the centre to the periphery if the pulsation theory is accepted, may detract from the accuracy of the method in actual practice. It appears, however, that a careful study of the elements of some Cepheid such as SU Cassiopeiæ, of which the radial velocity, magnitude and spectral type, and presumably therefore temperature, are known as functions of the time, and of which the parallax has been fairly accurately determined, should enable one

to decide definitely whether the pulsation hypothesis is admissible or not. If it is true, this method of determining parallaxes might be capable of sufficient accuracy in many cases to ascertain how exactly the well-known relation between period and absolute magnitude is fulfilled.

Farnborough:
1918 June 6.

The Short-period Variable RS Boötis. By C. Martin
and H. C. Plummer.

1. The variability of RS Boötis was discovered by Mrs. Fleming from a plate taken at Arequipa in 1906. This star* is B.D. +32°.2489, No. 142932 on the Harvard system, and its discovery number was 14.1907. The period of nine hours assigns it to the merihemeral group of physical variables.

2. An epoch of maximum was determined by Seares† at J.D. 2418 115.626. The corresponding epoch for the Dunsink observations is J.D. 2421 724.444, giving an interval of 3608.818 days. On the assumption that this contains 9564 periods, the length of a single period is 0^d.3773335, which agrees to six places with that determined by Pračka‡ about 1909. The elements adopted are

$$\text{J.D. } 2421\,724.444 + 0.377334 \text{ E.}$$

3. The magnitudes of the comparison stars have been derived from a single plate exposed at equal altitudes on the region of the variable and on the χ Persei cluster, for which Schwarzschild's magnitudes have been used. In this way the magnitudes m in Table I. were obtained and adopted in the reduction of the variable. The plate residuals give the small corrections incorporated in the column m' , the probable error for a single plate, and the extreme range shown in the last column.

TABLE I.

B.D. +32°.	m .	Corr.	m' .	P.E.	Range.
2485 (E)	9.62	-0.010	9.61	±0.014	.10
2485 (W)	10.28	+0.008	10.29	±0.022	.15
2487	10.75	+0.001	10.75	±0.015	.11
2488	11.24	-0.005	11.24	±0.018	.14

4. The results from forty-two exposures on the variable are given in Table II. In the second and third columns are recorded the astronomical date and G.M.T., together with the Julian day and decimal fraction, corresponding to the middle of each exposure, the length of which is shown in the fifth column. The phase in days,

* *H.C.O. Circular*, 124. *A.N.*, 4159, 4212.

† *Laws Bull.*, 15.

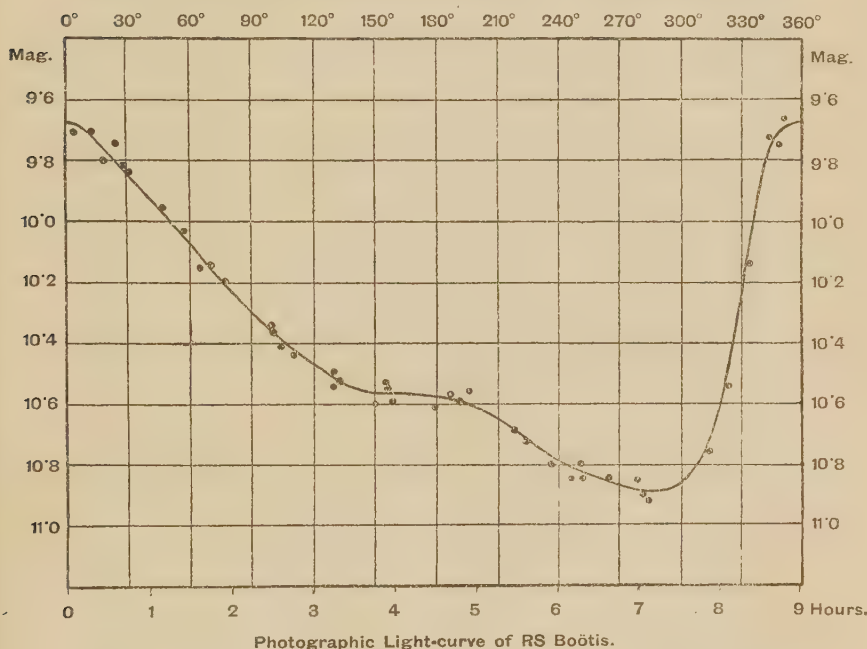
‡ *A.N.*, 4396.

given in the fourth column, is based on the elements adopted in § 2. The residuals in the seventh column are taken graphically from the comparison of the photographic magnitudes of the variable in the sixth column with the accompanying curve. They indicate a probable error of ± 0.018 for a single measured magnitude.

5. The curve shown in the figure and used in this comparison represents the equation

$$m = 10.41 + 0.45 \sin(\theta + 229^\circ) + 0.26 \sin(2\theta + 244^\circ) \\ + 0.11 \sin(3\theta + 277^\circ) + 0.07 \sin(4\theta + 297^\circ) \\ + 0.05 \sin(5\theta + 333^\circ) + 0.03 \sin(6\theta + 354^\circ),$$

where θ is reckoned from the assumed epoch of maximum, and its



period 2π corresponds with the period of the star. The regularity of the phases of the terms is remarkable, and can be traced through several later terms, though their coefficients are so small as to render them negligible. By a change of epoch the equation becomes

$$m = 10.41 - 0.45 \cos \phi + 0.26 \cos(2\phi - 124^\circ) + 0.11 \cos(3\phi - 50^\circ) + \dots$$

giving $\phi_2 = 124^\circ$, $\phi_3 = 50^\circ$ on the (ϕ_2, ϕ_3) diagram of Mr. Phillips. This is very near the mean position* of the merihemeral group,

* Cf. *M.N.*, 78, 161.

placing RS Boötis, like RR Lyræ ($118^{\circ}, 56^{\circ}$) and SU Draconis ($118^{\circ}, 58^{\circ}$), in the neighbourhood of the representative point ($118^{\circ}, 59^{\circ}$) of σ Ceti or of R Draconis ($122^{\circ}, 60^{\circ}$).

6. The nature of the corresponding visual curve can be inferred from the note by Seares, already quoted. "Of the interval of $9^h 3^m$ corresponding to the period, $1^h 21^m$ are required for the ascent from minimum to maximum." The photographic curve as drawn shows a rather longer time, but the absence of observations at the exact moment of change leaves this doubtful. The minimum being apparently flat, its exact instant must be very difficult to determine. The rise from minimum to maximum is certainly very rapid in this star.

"The decline covers about $4^h 50^m$." The end can be identified with the point where the photographic curve reaches the magnitude 10.6 and resumes a fairly rapid fall. "Throughout the remaining $2^h 52^m$ the light is sensibly constant, the variation during this interval amounting, at most, to 0.05 or 0.06. . . . The amplitude is one magnitude." Thus the visual curve shows a nearly stationary phase over at least a quarter of the whole period before the minimum is reached, while the photographic descends more than 0.2 in the same interval. This difference is nearly the same as the difference in total range, for the photographic amplitude is 1.2. Between maximum and minimum there is therefore an increase of 0.2 in the colour index, corresponding to a change of about half a spectral class, and it appears that the change is chiefly operative in the visual stationary phase. The failure to reproduce this feature photographically was at one time perplexing, but it is now easily understood that while the visual light may remain practically steady, type and colour can vary and change the actinic effect.

TABLE II.

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. - Curve.	Images.
	1918	h	m	2421		min.			
1	Apr. 2	10	44	686.449	.116	60	10.43	+ .01	Good.
2		2	11 46	686.490	.157	44	10.60	+ .04	Good.
3		2	12 31	686.521	.188	44	10.61	+ .03	Fair.
4		16	11 2	700.459	.164	50	10.54	- .02	Good.
5		16	12 1	700.501	.206	50	10.56	- .04	Good.
6		16	13 0	700.542	.247	50	10.80	+ .03	Good.
7		17	9 56	701.414	.365	50	9.74	+ .03	Fair.
8		17	10 54	701.455	.028	50	9.82	.00	Fair.
9		17	11 53	701.495	.068	50	10.15	+ .04	Fair.
10		17	12 46	701.532	.105	42	10.34	- .03	Good.
11		29	10 8	713.422	.298	40	10.92	+ .03	Good.
12		29	10 50	713.452	.328	40	10.76	+ .04	Good.

TABLE II.—*continued.*

Serial No.	Date G.M.T.	Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. — Curve.	Images.
	1918 h m	2421		min.			
13	Apr. 29 11 37	713'484	'360	40	9'72	— '04	Good.
14	29 12 18	713'513	'012	40	9'71	— '01	Good.
15	May 4 10 13	718'426	'019	40	9'80	+ '03	Good.
16	4 10 54	718'455	'048	40	9'96	'00	Good.
17	4 11 29	718'479	'072	18	10'15	'00	Fair.
18	6 10 17	720'429	'136	40	10'54	+ '03	Good.
19	6 11 3	720'460	'167	40	10'59	+ '02	Good.
20	6 11 43	720'489	'196	40	10'57	— '02	Good.
21	6 12 39	720'528	'235	40	10'72	'00	Good.
22	6 13 20	720'556	'263	40	10'84	+ '02	Good.
23	6 14 6	720'587	'294	40	10'89	'00	Good.
24	9 10 0	723'417	'105	40	10'36	'00	Good.
25	9 10 43	723'448	'136	40	10'49	— '03	Good.
26	9 11 24	723'475	'163	40	10'53	— '04	Good.
27	9 12 18	723'513	'201	40	10'60	'00	Good.
28	9 12 59	723'541	'229	40	10'69	'00	Good.
29	9 13 47	723'575	'263	40	10'80	— '02	Good.
30	9 14 28	723'603	'291	40	10'85	— '04	Fair.
31	10 9 59	724'416	'349	40	10'14	+ '03	Fair.
32	10 10 43	724'448	'004	40	9'71	— '03	Good.
33	10 11 24	724'475	'031	40	9'84	— '01	Good.
34	10 12 35	724'525	'081	40	10'20	— '01	Good.
35	10 13 16	724'553	'109	40	10'41	+ '02	Fair.
36	10 13 59	724'583	'139	29	10'52	'00	Fair.
37	11 10 57	725'457	'258	36	10'84	+ '04	Good.
38	11 11 25	725'476	'277	20	10'85	— '01	Fair.
39	16 10 36	730'442	'338	40	10'54	+ '03	Fair.
40	16 11 20	730'472	'368	40	9'67	— '02	Good.
41	16 12 9	730'506	'025	40	9'74	— '06	Fair.
42	16 12 58	730'541	'060	40	10'03	— '02	Fair.

The Eclipsing Variable W Ursæ Minoris. By C. Martin
and H. C. Plummer.

1. The variability of W Ursæ Minoris was discovered in 1913 by Mr Astbury, and also independently by Mr Davidson at Greenwich. The star is B.D. +86°.244, and received the different provisional numbers 12.1913 and 23.1913 in the two announcements.* As this system of temporary nomenclature has now fallen into abeyance, it may be hoped that it will never be resumed. If the provisional number is not used, it serves no purpose whatever. If it is used in the early observations, it only causes an unnecessary difficulty in tracing the references. Where a B.D. number is available, nothing more is wanted. For those variables which have no B.D. number a distinctive name is convenient, but once given, it should be permanent.

2. Greenwich photographic observations gave for the epoch of minimum J.D. 2419 487.850 + 1.7012 E. A minimum is indicated by the Dunsink observations at J.D. 2421 219.685, so that 1018 periods are comprised in an interval of 1731.835 days. The elements now adopted are therefore

$$\text{J.D. } 2421 \ 219.685 + 1.701213 \text{ E.}$$

3. The reductions have been made precisely as in similar work at this Observatory. Two plates were exposed at equal altitudes on the region of the variable and on the χ Persei cluster; Schwarzschild's magnitudes for stars in the latter region were used, and on these are based the magnitudes m_1 , m_2 of the four comparison stars in Table I. The mean magnitudes m have been adopted in the

TABLE I.

Star.	B.D. +86°.	m_1 .	m_2 .	m .	Corr.	m' .	P.E.	Range.
<i>a</i>	264	8.32	8.40	8.36	+ .017	8.38	± .017	.11
<i>b</i>	256	9.00	9.10	9.05	- .031	9.02	± .026	.17
<i>c</i>	263	9.30	9.36	9.33	+ .002	9.33	± .023	.17
<i>d</i>	254	9.86	9.98	9.92	+ .010	9.93	± .013	.13

reduction of the seventy-six plates of the variable. The residuals from these plates give the mean corrections in the sixth column, and hence the magnitudes m' , which should be more homogeneous than those adopted. The probable errors for a single plate and the extreme range are shown in the two last columns. The distribution of the comparison stars is extremely adverse to accurate measures of the variable. This star is situated near the middle of the E. side of the plate, while *a* and *c* are near the S.W. corner, and *b* and *d* are on the N. side and equally in the W. half of the plate. These circumstances are as unfavourable as possible, and will fully account for some irregularities which make the inter-

* *A.N.*, 4655, 4676.

pretation of the light-curve rather more uncertain than would have been the case if it had been possible to select comparison stars symmetrically distributed about the variable at the centre of the plate.

4. The derived magnitudes of W Ursæ Minoris are given in the sixth column of Table II. In the second and third columns are recorded the astronomical date and G.M.T., together with the Julian day and decimal fraction, corresponding to the middle of each exposure, the length of which is shown in the fifth column. The phase in days, given in the fourth column, is based on the elements of minimum adopted in § 2.

5. To the stationary phases twenty-eight observations refer and give a mean magnitude $8.75 \pm .005$, the probable error of a single observation being $\pm .026$. None of these measures is taken within $0^d.24$ of primary or secondary minimum. The whole variation is sensibly symmetrical, and a circular orbit may be assumed. Primary minimum can be taken at 9.64, giving a loss of $0^m.89$, or 0.56 of the total light. The secondary minimum is difficult to estimate, the uncertainty being entirely due to the indifferent quality of the measures, the cause of which has been explained above. But it cannot apparently be far from 8.88, giving a loss of $0^m.13$, or 0.113 of the total light.

6. The best starting-point is found by assuming that the variation is due to partial eclipses of uniform stars, and employing the methods of Russell and Shapley.* In Russell's notation it is then found that

$$\sin^2 \theta = 0.288\omega_2(n) + 0.175\omega_1(n)$$

represents the curve of primary minimum, whence

$$\chi(k, a_0, \frac{1}{4}) = C/D = 288/175 = 1.646.$$

In itself this gives a double range of solutions between the values 0.3 and 0.5 of $k = r_2/r_1$. But the possibilities in this direction are very restricted, because even the total eclipse, corresponding to $k = 0.47$, which is the limit of one series, gives only a loss of $0^m.11$ at the secondary minimum.

7. On the assumption that the eclipse at primary minimum is just total, with the given value of k , a uniform solution is easily obtained. It is found that

$$\sin^2 \theta = 0.1384 + 0.0978\psi(k, a_1),$$

and hence that the inclination of the orbit is

$$i = 74^\circ.5,$$

and its radius being unity, the radii of the stars are

$$r_1 = 0.503, \quad r_2 = 0.237.$$

The light of the smaller, brighter star (mag. 9.38) being 0.56 ,

* *Ap. J.*, 35, 36.

that of the larger (mag. 9^m64) is 0[·]44, and the ratio of surface brightness is

$$J_b/J_f = 14/11(0\cdot47)^2 = 5\cdot76.$$

If the components have equal mass, the densities in terms of the Sun are 0[·]018 for the larger and 0[·]176 for the smaller; and if the masses are both of the same order as the Sun, the radii in terms of the Sun's radius are about 3[·]8 and 1[·]8, the separation of the centres being in the same unit about 7[·]5. This solution gives rather too small a depth at secondary minimum, and perhaps too flat a curve at primary minimum. But probably no better representation is possible with uniform stars.

8. Darkening towards the limb is certainly more likely, especially with photographic observations of magnitude. Now to the above solution corresponds a solution with darkened elements, giving directly $k = 0\cdot65$. This would probably give a good representation of the primary minimum. But it involves the loss of at least half the light of the larger star, or 0[·]22, at secondary minimum. This means a depth of 0^m27, which is certainly too great. A total eclipse at primary minimum, with stars completely darkened at the limb, can therefore be rejected.

9. There remains the alternative hypothesis of the partial eclipse of darkened stars, and this is the most probable explanation. Trial shows that the appropriate value of k is about 0[·]55, and that primary minimum is represented by a loss of the fraction (a_0) 0[·]9 of the light of the brighter and smaller component. This means that the light of the brighter star is 0[·]622, of the fainter 0[·]378, and that the individual magnitudes are 9[·]27, 9[·]81. It easily follows that the radii of the stars are

$$r_1 = 0\cdot476, \quad r_2 = 0\cdot262$$

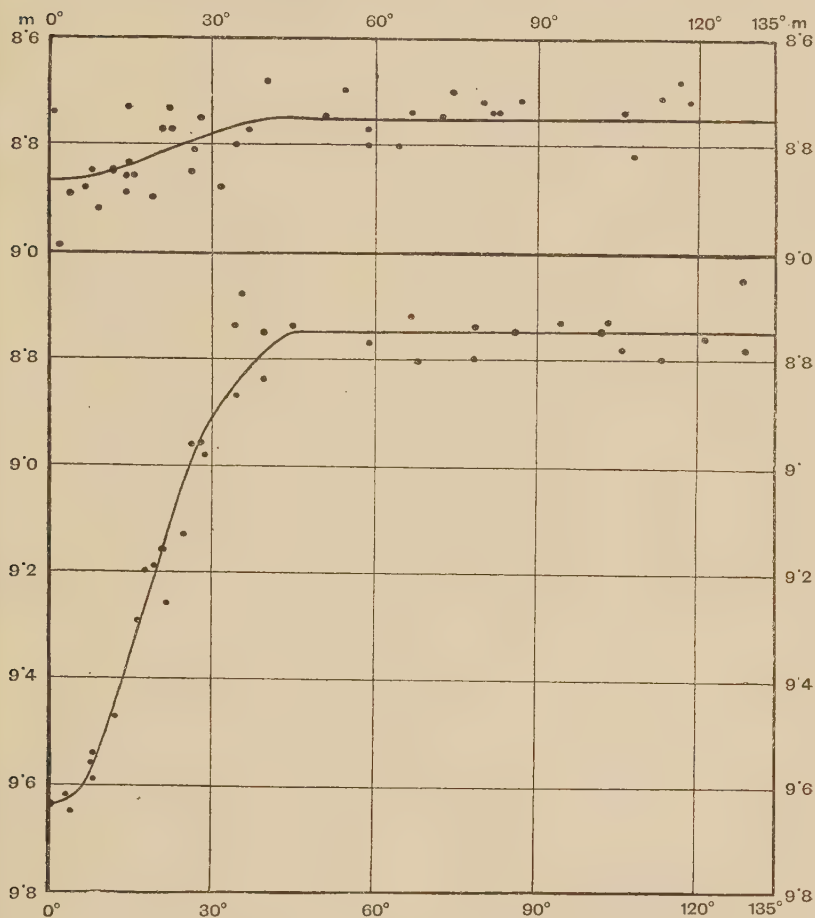
in terms of the orbital radius, and that the inclination of the orbit is

$$i = 72^\circ\cdot3.$$

The densities in terms of the Sun, on the assumption of equal masses, are 0[·]022 and 0[·]130. If the masses are, further, equal to that of the Sun, the radii of the components become 3[·]6 and 2[·]0 in terms of the Sun's radius, and the radius of the orbit is 7[·]5 when expressed in the same unit. The corresponding depth at secondary minimum is 0^m12. According to the observations it should perhaps be a little more. This is uncertain, but it could easily be obtained by slightly increasing the radius of the smaller star without producing any other appreciable effect. Were the observations much more accurate than they are, it would be difficult to estimate exactly the limiting phases of the partial eclipses and at the same time the degree of darkening towards the limb. On the whole, it is rather surprising that the indications are as precise as they seem to be.

10. The arc of eclipse appears to be just about a quadrant.

All the observations are plotted in the accompanying figure, in which the upper section shows the secondary minimum with the following stationary phase, and the lower section the primary minimum with the stationary phase following it. For both minima



Photographic Light-curve of W Ursæ Minoris.

the observations of the descending branch have been reflected on the ascending branch. This process conceals any tendency there may be to want of symmetry between the two branches, but the observations would not justify any reliance on apparent differences. Both branches have therefore been combined by reflexion in the discussion. The discordances shown in the seventh column of Table II. are taken from the figure, and the curve represents partial eclipses of darkened stars. The results for a total eclipse

with uniform surfaces would be practically the same. The elements on either hypothesis can only be regarded as approximate, but no other orbit of this star seems to have been published. It appears a suitable object for good photometric observations.

TABLE II.

Serial No.	Date G.M.T.	Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1916	h m	2420	min.			
1	Apr. 4	9 36	958'400	0'702	42	8'88	+ '10 Good.
2		5 9 12	959'383	1'685	44	9'62	- '04 Fair.
3		5 10 3	959'419	0'020	44	9'65	+ '03 Good.
4		5 10 56	959'456	0'057	44	9'47	+ '02 Good.
5		5 11 49	959'492	0'093	44	9'19	- '01 Good.
6		5 12 42	959'530	0'131	44	8'96	'00 Fair.
7		5 13 35	959'567	0'168	44	8'68	- '07 Good.
8		7 9 6	961'379	0'279	39	8'77	+ '02 Good.
9		7 11 17	961'470	0'370	44	8'80	+ '05 Good.
10		7 12 10	961'507	0'407	42	8'75	'00 Good.
11		7 13 5	961'546	0'446	44	8'73	- '02 Good.
12		7 13 58	961'583	0'483	44	8'75	'00 Good.
13		26 10 4	980'420	0'606	44	8'65	- '10 Fair.
14		26 10 55	980'455	0'641	44	8'75	'00 Good.
15		26 11 48	980'492	0'678	44	8'77	+ '02 Fair.
16		28 10 5	982'421	0'906	44	8'85	'00 Good.
17		28 10 54	982'455	0'940	44	8'90	+ '08 Good.
18		28 11 47	982'491	0'976	44	8'81	+ '02 Good.
19		28 12 40	982'528	1'013	44	8'80	+ '05 Fair.
20	May	1 10 2	985'418	0'501	44	8'78	+ '03 Good.
21		1 10 52	985'453	0'536	44	8'80	+ '05 Good.
22		1 11 45	985'490	0'573	44	8'76	+ '01 Good.
23		1 12 38	985'527	0'610	44	8'78	+ '03 Good.
24		3 10 15	987'427	0'808	44	8'92	+ '06 Fair.
25		3 11 7	987'462	0'843	44	8'99	+ '12 Good.
26		3 12 0	987'500	0'881	44	8'88	+ '02 Good.
27		3 12 52	987'536	0'917	44	8'89	+ '05 Good.
			2421				
28	June	1 12 18	016'513	0'974	44	8'85	+ '05 Good.
29		2 12 11	017'507	0'266	44	8'80	+ '05 Good.
30		6 12 17	021'512	0'869	44	8'89	+ '03 Fair.
31	Dec. 20	11 26	218'476	0'488	40	8'73	- '02 Bad.
32		21 11 56	219'497	1'513	46	8'75	'00 Fair.
33		21 13 39	219'570	1'586	40	9'13	+ '08 Bad.
34		21 14 33	219'606	1'622	46	9'29	- '02 Fair.
35		21 15 30	219'646	1'662	46	9'54	- '02 Fair.
36		21 16 25	219'685	0'000	46	9'64	'00 Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs.—Curve.	Images.
	1917	h	m	2421		min.			
37	Jan. 23	9	33	252'380	0'372	46	8'74	- '01	Fair.
38	Feb. 9	8	5	269'337	0'315	50	8'72	- '03	Fair.
39		14	8	274'336	0'212	39	8'74	- '01	Fair.
40		15	7	275'326	1'202	48	8'70	- '05	Fair.
41		15	8	275'365	1'241	46	8'74	- '01	Fair.
42		15	11	275'476	1'352	48	8'74	- '01	Fair.
43		15	12	275'510	1'386	35	8'71	- '04	Bad.
44	Mar. 12	10	6	300'421	0'779	46	8'86	+ '02	Fair.
45		13	10	301'441	0'098	29	9'16	'00	Fair.
46		14	10	302'435	1'092	44	8'75	'00	Fair.
47		14	11	302'472	1'129	46	8'77	+ '02	Fair.
48		14	12	302'511	1'168	44	8'74	- '01	Fair.
49		16	10	304'454	1'410	44	8'72	- '03	Fair.
50		26	9	314'407	1'155	44	8'80	+ '05	Good.
51		26	10	314'446	1'194	44	8'75	'00	Good.
52		26	11	314'481	1'229	44	8'72	- '03	Good.
53	Apr. 12	9	25	331'393	1'129	44	8'80	+ '05	Fair.
54		12	12	331'503	1'239	44	8'74	- '01	Fair.
55		16	10	335'452	0'084	44	9'20	- '05	Good.
56		16	11	335'493	0'125	44	8'96	- '03	Fair.
57		16	12	335'529	0'161	44	8'74	- '12	Good.
58		19	10	338'430	1'361	32	8'82	+ '07	Good.
1918									
59	Feb. 5	9	33	630'398	0'720	44	8'75	- '04	Good.
60		5	10	630'431	0'753	31	8'77	- '05	Fair.
61		5	11	630'462	0'784	44	8'86	+ '02	Fair.
62		7	10	632'419	1'040	50	8'68	- '08	Fair.
63		9	11	634'482	1'402	40	8'68	- '07	Fair.
64	Mar. 6	8	16	659'344	0'746	36	8'77	- '04	Fair.
65		6	9	659'381	0'783	40	8'73	- '11	Fair.
66		6	10	659'451	0'853	40	8'74	- '13	Fair.
67		6	11	659'485	0'887	40	8'85	- '01	Fair.
68		6	12	659'518	0'920	30	8'84	'00	Bad.
69		12	8	665'365	1'663	44	9'56	- '01	Fair.
70		12	10	665'441	0'038	44	9'59	+ '02	Fair.
71		13	8	666'358	0'955	44	8'73	- '08	Fair.
72	Apr. 1	9	4	685'378	1'262	25	8'72	- '03	Fair.
73	May 7	10	35	721'442	1'600	40	9'26	+ '12	Fair.
74		29	11	743'471	1'514	30	8'84	+ '05	Fair.
75		29	11	743'496	1'539	30	8'87	+ '02	Fair.
76		29	12	743'522	1'565	30	8'98	+ '04	Fair.

Note on the Variable B.D. + 56°.547. By H. C. Plummer, M.A.

In *Monthly Notices*, 77, 651, it was shown that the variable B.D. + 56°.547, discovered by Mr. J. Van der Bilt, had repeated its changes with remarkable fidelity in two consecutive periods of 700 days, reckoned from J.D. 2418,500. In order to decide whether this character could be traced before and after the four years covered by the Dunsink observations, an appeal was made to Professor E. C. Pickering for observations made at other dates. With his invariable courtesy he promptly sent a set of measures very kindly made by Miss Leavitt. These left no doubt that the repetition noticed was not a permanent feature. The measures agreed very fairly with the Dunsink observations, where the dates made a comparison possible. Professor Pickering remarks that "the object is a difficult one owing to its red color and small range," and Miss Leavitt notes that "on account of the redness of the variable the character of the image differs greatly from that of the white stars in the sequence." The comparisons at Harvard were therefore made with two stars having the same class of spectrum as the variable, K5. There can be no doubt that achromatism gives the reflector a great advantage in variable star work.

A few recent observations obtained by Mr. Martin are appended to this note. They can be readily compared with the results shown in the diagrams (77, 652) by subtracting 2100 or 2800 days from the dates given here. When this is done, the failure of the star to repeat its earlier changes is quite obvious. B.D. + 56°.547 must therefore be classed as a distinctly irregular variable.

The repetition of the variations after an interval of two years remains nevertheless a fact, and a singular one, though apparently accidental. It is not proposed to continue the observations at this Observatory now that the character of the object is definitely established.

Date G.M.T.	Julian Day.	Exposure.	Magn.	Images.
1917 h m	2421	min.		
Sept. 11 12 35	483'525	40	10'19	Good.
20 12 0	492'500	40	10'19	Good.
Nov. 7 11 33	540'481	40	10'32	Good.
Dec. 10 10 28	573'437	40	10'38	Fair.
1918				
Jan. 1 8 28	595'353	60	10'38	Good.
8 8 6	602'337	60	10'38	Good.
16 8 24	610'350	40	10'34	Fair.
Feb. 5 8 30	630'354	50	10'06	Good.
Mar. 12 9 38	665'402	44	10'01	Fair.
13 9 28	666'395	44	9'98	Bad.
Apr. 2 9 21	686'389	60	10'29	Good.
15 10 47	699'450	60	10'30	Fair.
May 7 11 31	721'479	50	10'33	Fair.

The Parallaxes of the B Stars which lie between the Limits of 4^h to 8^h R.A. and +24° to +32° Declination. By Sir F. W. Dyson and W. G. Thackeray.

The proper motions of the stars between +24° and +32° declination down to the ninth magnitude have been recently determined at Greenwich, and will be published with the positions of these stars as determined from the Greenwich Observations 1906-1915. Owing to the war this publication has been delayed, but one advantage arises from this delay, that it will be possible to give with the proper motions the spectral types of the stars as determined by Miss Cannon, and now being published in the Harvard Catalogue of Stellar Spectra. By the kindness of Professor Pickering a large part of the material which it is proposed to incorporate has already been communicated.

The zone 24° to 32° N. Declination cuts the galactic plane at 5^h 30^m R.A., and B stars extend from a little before 4^h to 8^h. There is then a gap in which there are practically no B stars till the zone cuts the Milky Way again. The stars near 6^h have a large parallactic factor almost wholly in declination. On the assumption that the stars have no systematic motion apart from this, the mean parallaxes of B stars of different magnitudes are determinable.

A list of the B stars and their proper motions is given at the end of this paper. From this the following table has been derived, to which the stars of type A 0 have been added:—

Proper Motions in Declination (Unit "0001).

Type.	-5 ^m .9.			6 ^m .0-6 ^m .9.			7 ^m .0-7 ^m .9.			8 ^m .0-		
	Mag.	p.m.	No.	Mag.	p.m.	No.	Mag.	p.m.	No.	Mag.	p.m.	No.
B 0							7.45	-120	1			
B 2	4.90	-40	1	6.86	+210	1	7.70	+10	1	8.20	0	1
B 3	5.25	-210	2	6.02	-80	1	7.59	-192	3	8.10	+50	1
B 5				6.28	-25	2	7.47	+45	4	8.42	-137	9
B 8	5.83	-180	3	6.56	-206	5	7.72	-63	6	8.49	-51	22
B 9	5.73	-270	6	6.50	-206	10	7.55	-87	13	8.45	-70	47
A 0	5.32	-283	7	6.45	-204	8	7.58	-132	37	8.67	-101	160

If the apex of the solar motion be taken at 270°+35°, the parallactic factor in declination is .88; while if it be taken at 270°+25°, the factor is .81. Taking the solar motion as 20 kilometres a second, the mean parallaxes are obtained by dividing by 3.71 or 3.42, according to the positions assumed for the apex.

The mean intrinsic magnitude M is computed from these by the formula

$$M = m + 5 + 5 \log \varpi,$$

and is the magnitude corresponding to the parallax 0".1.

For the stars of types B 8 and B 9 the following values of ϖ_1 and M_1 (apex assumed to be at $270^\circ + 35^\circ$) and of ϖ_2 and M_2 (apex assumed to be at $270^\circ + 25^\circ$) are found:—

Parallaxes and Intrinsic Magnitudes of Stars of Types B 8 and B 9.

<i>m.</i>	μ_8	ϖ_3	ϖ_2	M_1	M_2	No.
1.78	"1770	"0479	"0518	+0.2	+0.4	1
5.76	'0249	'0067	'0073	-0.1	+0.1	9
6.52	'0206	'0056	'0060	+0.2	+0.4	15
7.61	'0079	'0021	'0023	-0.7	-0.5	19
8.46	'0066	'0018	'0019	-0.3	-0.1	69

Parallaxes and Intrinsic Magnitudes of Stars of Type A 0.

<i>m.</i>	μ_8	ϖ_1	ϖ_2	M_1	M_2	No.
5.32	'0283	'0076	'0088	+0.7	+0.9	7
6.45	'0204	'0055	'0060	+1.1	+1.3	8
7.58	'0132	'0036	'0039	+1.3	+1.5	37
8.67	'0101	'0027	'0030	+0.8	+1.0	160

The distances of the B stars has been considered by Kapteyn, Plummer, Charlier, Gyllenberg, Malmquist. Their researches are mainly confined to bright stars, whose radial velocities have been determined or whose proper motions are given in Boss's Preliminary General Catalogue. The extension to fainter stars is difficult on account of the smallness of the proper motions, and makes a severe demand on freedom from systematic and accidental error. The values obtained for M indicate that the faint stars of types B 8, B 9, and A 0 confirm the view that stars of these types have not a great range in absolute luminosity. No stress can be laid on the want of regularity in the results obtained for magnitudes $6^m.5$, $7^m.5$, and $8^m.5$. From the catalogue given in Kapteyn's paper, the mean absolute magnitude of 69 stars of types B 8 and B 9 is found to be $0^m.55$, and 56 of these are between $-1^m.0$ and $+1^m.9$. Malmquist finds

B 8	-20	from	276	stars.
B 9	-15	„	140	„
A 0	$+40$	„	1700	„

The following list gives the approximate co-ordinates, proper motions, magnitude and type of spectrum, and galactic latitude of the B stars used in this paper (the * denotes that the star is in Boss's Catalogue):—

List of B Stars.

Approximate.				Proper Motion.		Harvard Magnitude.	Harvard Spectral Type.	Galactic Latitude.
R.A.		Dec.	1910.0.	R.A.	Dec.			
				(Unit "1001.)				
h	m	s	°			m		°
4	7	3	31 34	+ 8	- 1	8.4	B 9	- 14
	15	1	25 37	+ 8	+ 11	7.81	B 9	- 16
	15	11	31 2	+ 41	- 5	8.2	B 9	- 13
	17	6	25 25	+ 25	- 32*	5.38	B 9	- 16
	18	34	24 6	+ 17	- 23*	6.16	B 8	- 16
	27	8	31 36	- 23	- 6	8.6	B 9	- 10
	29	0	28 46	+ 5	- 22*	5.70	B 9	- 12
	32	3	30 14	+ 19	- 5	8.56	B 9	- 11
	32	29	31 49	+ 12	- 21	7.08	B 3	- 10
	35	37	25 49	- 9	- 31	8.4	B 9	- 12
	42	41	29 37	- 17	- 24	7.36	B 9	- 9
	45	14	28 10	+ 3	- 17	7.72	B 3	- 9
	52	39	24 55	+ 30	- 60*	5.65	B 9	- 10
	53	24	24 21	+ 12	- 9	8.0	B 5	- 10
	54	17	27 11	+ 3	- 49	6.59	B 9	- 8
	55	44	26 24	- 12	+ 21	7.8	B 5	- 9
	55	58	31 39	- 5	- 5	7.50	B 9	- 6
	57	16	24 11	- 13	- 31	8.4	B 9	- 9
5	0	19	26 18	+ 1	- 8	6.56	B 5	- 8
	2	37	24 9	+ 3	- 11*	5.50	B 3	- 9
	7	33	29 52	- 20	- 20	9.1	B 9	- 4
	9	13	31 38	+ 5	+ 7	8.2	B 8	- 9
	10	18	31 43	+ 4	- 4	8.4	B 9	- 3
	13	52	28 41	+ 56	- 37	9.1	B 9	- 5
	15	27	27 52	- 25	- 30*	6.30	B 9	- 4
	17	16	29 39	- 4	- 29	7.76	B 9	- 3
	17	41	28 51	+ 25	- 30*	6.39	B 9	- 5
	18	50	31 4	- 12	- 9*	5.93	B 9	- 2
	19	33	30 5	+ 7	- 4	8.76	B 8	- 2
	20	30	29 54	+ 9	- 3	8.06	B 9	- 2
	20	36	28 32	+ 31	- 177*	1.78	B 8	- 3
	21	23	30 8	0	- 19*	5.72	B 9	- 2
	21	40	31 19	+ 8	+ 6	8.4	B 8	- 1
	26	50	26 55	- 25	- 11	7.11	B 5	- 3
	27	10	26 41	+ 48	0	8.2	B 2	- 2
	27	59	24 34	0	- 42	6.92	B 8	- 3
	28	59	27 59	+ 9	+ 20	8.0	B 8	- 0

List of B Stars—continued.

Approximate.		Proper Motion.		Harvard Magnitude.	Harvard Spectral Type.	Galactic Latitude.
R.A. 1910.0.	Dec.	R.A. (Unit "001.)	Dec.			
h m s	° ' "			m		°
5 30 27	26 13	+ 7	- 3	8.5	B 8	- 2
31 0	27 51	+36	+ 7	8.6	B 8	+ 1
31 31	26 52	+13	-30	5.70	B 8	- 2
33 17	28 24	+ 8	-12	8.2	B	0
33 36	29 10	+37	+ 3	6.00	B 5	- 1
33 37	30 50	+ 1	+13	7.52	B 5	+ 1
34 10	25 51	+31	-31*	5.00	B 3	- 1
34 31	31 51	- 3	- 9	8.0	B 9	+ 1
34 46	31 19	- 5	- 1	5.96	B 8	+ 1
36 0	26 34	+12	+36	8.0	B 9	- 1
36 23	27 44	+11	+ 7	8.5	B 9	0
38 6	25 24	+15	+21	6.86	B 2	- 1
38 11	27 15	-11	+23	7.8	B 9	0
38 20	29 28	+ 1	-39	8.2	B 5	+ 2
38 17	30 54	+ 7	- 3	8.1	B 8	+ 1
38 32	29 59	-12	+ 7	8.11	B 9	+ 1
38 55	28 59	+ 3	-13	8.1	B 5	+ 1
39 20	26 18	-19	+11	7.22	B 9	0
39 50	30 8	+ 3	-10	8.24	B 5	+ 1
39 55	28 39	+59	+21	8.8	B 9	+ 1
40 20	25 53	+28	+ 7	7.7	B 9	0
41 26	29 24	+ 3	+ 8	8.0	B 9	+ 1
41 32	28 35	+ 3	- 9	8.1	B 9	+ 2
42 11	28 45	- 9	+18	8.7	B 8	+ 1
43 0	28 18	+13	-24	8.5	B 8	+ 1
43 10	27 20	- 3	+ 4	8.1	B 8	+ 1
44 13	30 32	+39	-39	8.8	B 5	+ 2
44 40	31 2	+25	-19	8.0	B 5	+ 3
45 6	30 42	+29	+ 4	9.4	B 5	+ 3
46 25	29 54	- 7	-21	8.91	B 9	+ 3
46 45	31 26	+ 4	-12	8.8	B 9	+ 3
47 17	28 26	+ 8	+ 4	9.0	B 5	+ 2
47 30	26 25	+27	+ 5	8.1	B 3	+ 2
48 3	24 16	- 7	-20	7.8	B 8	0
48 10	28 56	- 4	- 7	9.0	B 9	+ 2
48 24	26 24	- 7	+ 3	8.4	B 8	+ 1
48 29	30 29	-31	- 5	7.46	B 5	+ 3

List of B Stars—continued.

Approximate.			Proper Motion.		Harvard Magnitude.	Harvard Spectral Type.	Galactic Latitude.
R.A.	Dec.		R.A.	Dec.			
1910.0.			(Unit "001.)				
h m s	° ' "				m		°
5 49 16	28 22		+40	+28	8.6	B 9	+ 2
49 49	30 42		+ 5	+ 5	8.4	B 8	+ 3
50 0	27 42		-27	+ 1	7.7	B 2	+ 2
50 45	29 44		+ 5	-32	8.6	B 8	+ 3
51 25	24 14		-17	- 8	6.02	B 3	+ 1
52 5	24 36		-24	-17	7.31	B 8	+ 1
52 24	25 57		- 1	- 4*	4.90	B 2	+ 2
52 26	28 11		+15	+ 8	8.8	B 8	+ 3
52 29	24 36		+ 9	-10	7.96	B 8	+ 1
52 39	29 37		+55	-13	7.81	B 9	+ 4
53 14	31 1		-12	-13	9.4	B 8	+ 5
53 15	27 36		+25	+ 5	8.2	B 9	+ 3
53 26	27 33		-25	- 1	7.9	B 9	+ 3
55 21	27 34		-23	-20	6.08	B 8	+ 3
57 17	31 3		-13	+ 1	9.0	B	+ 5
57 57	26 32		- 1	+ 4	6.97	B 8	+ 3
58 18	28 39		+51	-15	8.8	B 9	+ 4
54 0	28 59		+35	- 6	8.6	B 9	+ 4
59 38	24 21		+24	-18	8.8	B 9	+ 3
6 0 23	28 57		+25	- 9	7.45	B 0	+ 5
0 31	30 47		+25	- 3	8.5	B 8	+ 6
0 51	27 4		+25	- 7	8.6	B 8	+ 4
1 43	26 42		+15	-23	7.01	B 9	+ 4
2 0	27 52		+29	- 2	8.6	B 9	+ 3
2 25	25 28		-29	+ 4	8.0	B 9	+ 4
3 5	25 40		- 4	+15	8.7	B 9	+ 4
3 26	24 22		- 3	-23	8.6	B 8	+ 3
5 32	28 56		+ 7	+ 8	7.63	B 8	+ 6
5 33	24 54		+11	- 5	8.21	B 9	+ 4
6 23	29 42		-17	+ 6	8.21	B 9	+ 6
9 41	29 15		+24	-16	8.6	B 9	+ 7
10 34	30 7		+16	-25	8.7	B 9	+ 7
13 10	25 23		+19	-14	8.0	B 8	+ 4
13 39	27 1		+ 7	- 3	8.5	B 9	+ 5
15 3	25 14		-11	-29	7.16	B 9	+ 4
16 45	25 38		+24	-14	8.6	B 9	+ 5
17 31	27 14		+15	-17	8.2	B 9	+ 6

List of B Stars—continued.

Approximate.		Proper Motion.		Harvard Magnitude.	Harvard Spectral Type.	Galactic Latitude.
R.A. 1910.0.	Dec.	R.A. (Unit "001.)	Dec.			
h m s	° ' "			m		°
6 17 41	28 48	+19	-27	8.4	B 8	+ 6
18 14	26 9	+35	+ 7	8.7	B 8	+ 5
18 30	25 28	+31	- 6	8.5	B	+ 5
19 8	29 45	+ 3	-26	6.52	B 9	+ 9
30 0	28 20	- 1	+11	8.0	B 9	+10
33 24	29 49	-12	+13	8.16	B 9	+11
33 53	28 21	-11	-23*	5.84	B 8	+11
35 7	30 18	+ 5	0	8.0	B 9	+12
36 43	31 54	+ 7	-34	8.2	B 9	+13
36 49	29 28	+26	- 9	7.46	B 9	+12
38 35	30 36	+17	-11	8.2	B 9	+12
38 48	30 35	-31	-21	8.2	B 9	+13
44 17	29 39	+15	-29	8.7	B 9	+14
49 3	29 56	+17	- 2	7.91	B 8	+15
50 8	24 7	0	-42	7.67	B 3	+13
50 19	27 24	+ 3	- 6	6.97	B 9	+14
53 55	26 2	-15	-14*	6.29	B 9	+14
56 1	26 42	-17	-16	8.5	B 9	+15
59 51	29 39	+16	-17	8.4	B 9	+17
7 1 46	28 19	+23	+ 9	6.23	B 9	+16
8 57	24 52	- 1	-16	6.66	B 9	+17
10 21	29 25	+27	-44	8.0	B 8	+19
11 43	31 52	+ 7	-22	6.68	B 8	+20
12 19	31 7	+ 1	-20	5.98	B 9	+20
17 54	25 41	- 5	- 2	9.0	B 9	+19
28 51	28 36	+ 3	+ 3	7.7	B 8	+20
31 46	31 49	-21	-33	7.60	B 9	+24
40 22	26 57	- 8	- 2	8.0	B 5	+24
43 43	30 57	-21	-30	8.7	B 9	+26
45 50	30 46	+13	- 2	8.0	B 9	+26
52 17	30 42	+11	-42	8.8	B 9	+28
0 6	27.47	+ 9	-19	6.16	B 9	+28

On the Corrections of Double Star Orbits determined graphically, and an Application to a Preliminary Orbit of Sirius. By R. Jonckheere.

(Communicated by the Astronomer Royal.)

The two last published orbits of Sirius were computed by Dobereck in 1903* and Lohse in 1908.† The last ten years have furnished new observations which, as indicated in my paper last month,‡ show that the computed places have lately systematically departed from the observations. These orbits are, however, already so good, as far as double star orbits go, that more than usual care is needed for a new determination.

Taking the elements of Dobereck's orbit, which after fifteen years still best represents the observations, I plotted down computed places conveniently distributed all round the orbit where there was a direct comparison with some of the best observations, such as those obtained with the 36-inch and 40-inch refractors of the Lick and Yerkes observatories. Having thus marked the computed and observed places, I drew a square round the first points and circles round the others. Two ellipses were then drawn, one bisecting all the squares representing the computed places, and the other representing as correctly as possible the positions indicated by the circles. This gave two very similar ellipses, the second, which I call the corrected orbit, being drawn in close broken lines for distinction.

I then used the admirable method of Zwiers and Russell§ to determine the true elements from those two apparent orbits. The construction lines were made simultaneously, those of the corrected orbit always being represented by the broken lines. The accompanying figure shows the construction. This is, however, not equal to the original drawing, as the lines had to be made thicker for reproduction. On the drawing, which measured 18 inches across, the construction was made on both sides of the figure, which thus gives very great accuracy for the determination of the elements.

By dealing with two orbits simultaneously, one of which is derived from computed places, we have as a thorough check of the whole construction simply to compare the elements determined with those already known. This comparison gave the following corrections to the graph :

$$\begin{aligned}\Delta e &= + 0.003 \\ \Delta \omega &= - 0^{\circ} 6' \\ \Delta i &= + 0^{\circ} 4' \\ \Delta \Omega &= + 0^{\circ} 7' \\ \Delta a &= + 0''.01\end{aligned}$$

* *Ast. Nach.*, 3981.

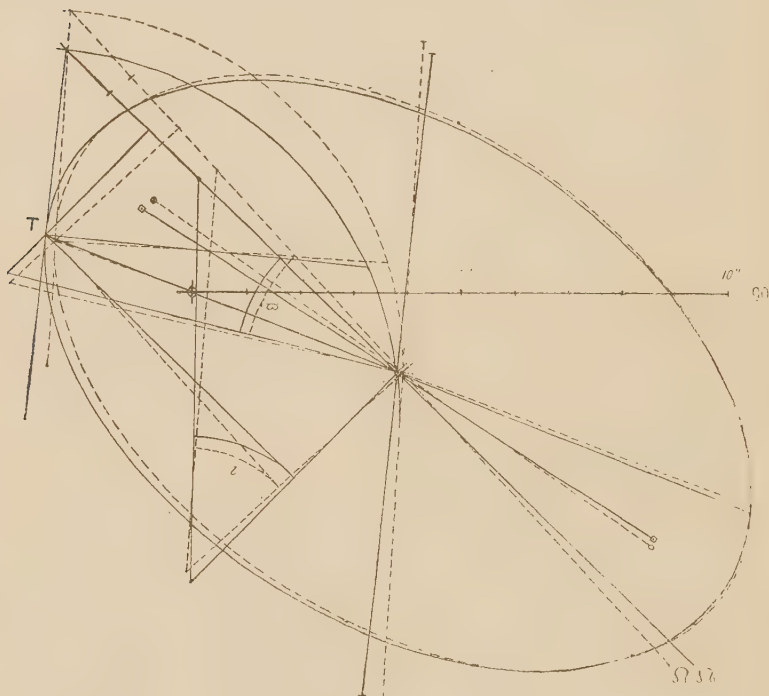
† *Potsdam Publications*, 1908, 20.

‡ *M.N.*, 78, 480.

§ *Ast. Nach.*, 3336, and *Pop. Ast.*, 6, 149.

This result shows the great accuracy it is possible to obtain by following the method of Prof. H. N. Russell.

The object, however, was not the absolute determination of the elements of the new orbit. The paper may move or stretch unequally, or again, the protractor may not be absolutely true; but it was thought that the small arcs between the construction lines of elements of the two orbits could be measured with



accuracy. In this manner we obtain in one solution the differential corrections of the geometrical elements required to satisfy to their best advantage the whole of the residuals in θ and ρ .

$$\Delta e = +0.013$$

$$\Delta \omega = +4^{\circ} 30'$$

$$\Delta i = +0^{\circ} 4'$$

$$\Delta \Omega = -3^{\circ} 7'$$

$$\Delta a = +0''.04$$

This is a result which, to have obtained analytically, would have involved a great deal of work, and if, on the other hand, position angles alone were used, as is the common practice to lessen the analytical work, the complete graphical solution has a

few points in its favour, especially in the case of very eccentric orbits such as that of Sirius.

Having thus obtained all the geometrical elements of the new orbit, it was an easy matter to find the period and time of periastron passage which, with these elements, best represented the observations. The result gave for the elements of a new provisional orbit:

$$T = 1894.10$$

$$P = 50.00$$

$$n = -7^{\circ}.20$$

$$e = 0.60$$

$$\omega = 214^{\circ}.4$$

$$i = 43^{\circ}.4$$

$$\Omega = 42^{\circ}.7$$

$$a = 7''.55$$

It will be seen that the period of fifty years given as a first approximation last month was adhered to.

With these elements the following residuals ($o - c$) were computed from all the complete series of observations extending over three or more nights. I have grouped these residuals separately under each observer, as in the accurate measurement of such a difficult object as the companion of Sirius, the size of the instrument and the personality of the observer are very prominent. The latitude should also be an important factor. Sirius is a much more difficult object at latitude 51° than it is at 37° or -26° .

<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>	<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>
Bond (15 in.).				Secchi (9 in.).			
1862.19	+0.6	+0.43	6	1865.22	-1.3	-0.89	8
				66.29	-3.2	-0.59	3
Rutherford.				71.16	+0.8	-0.57	3
1862.20	+1.1	+0.44	5				
63.21	-0.1	-0.41	6	Foerster.			
				1865.23	+1.0	+0.29	4
Marth.				Newcomb (26 in.).			
1864.14	+0.2	+0.39	3	1866.23	+0.2	-0.12	3
O. Struve (15 in.).				74.15	-0.2	+0.27	8
1864.22	+0.1	+0.66	3	Searle (15 in.).			
66.20	+2.2	+0.25	3	1867.10	+1.0	-0.18	5
69.23	+0.2	-0.20	3				
70.22	-0.2	+0.05	4	Winlock (15 in.).			
73.22	-0.8	-0.09	3	1867.19	-0.2	+0.28	6
79.22	-3.3	+0.53	3	70.17	-1.0	-0.21	5

<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>	<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>
	Auwers.				Dunér (9 in.).		
1868'22	- 1 ^o 3	- 0 ^{''} 50	5	1875'19	- 0 ^o 5	- 0 ^{''} 43	4
	Vogel.				Holden (15 in.).		
1868'23	- 0'3	+ 0'22	7	1875'21	- 1'8	+ 0'63	4
69'15	+ 4'7	+ 0'08	3	76'09	- 1'0	+ 0'66	6
70'24	- 1'2	+ 0'79	5	77'16	1'0	+ 0'33	4
	Bruhns.			78'07	- 1'4	+ 0'26	4
1868'24	- 1'0	+ 0'32	5	79'10	- 2'0	+ 0'69	6
	Engelmann (7 in.).			80'10	- 0'5	+ 0'24	4
1868'26	+ 1'1	- 0'08	5		Russell (11 in.).		
	Brünnow.			1876'14	- 0'8	+ 0'39	4
1869'10	+ 5'7	- 0'89	4		Burnham (18-36 in.).		
	Peirce (15 in.).			1877'97	- 0'2	+ 0'03	8
1870'13	+ 0'9	- 0'10	4	79'05	+ 0'7	- 0'13	10
72'10	+ 0'1	+ 0'16	5	80'11	+ 0'7	- 0'25	10
	Pechüle.			81'07	+ 0'8	- 0'20	8
1871'25	- 4'8	+ 0'78	3	81'99	+ 0'5	- 0'25	11
	Börger.			83'10	0'0	- 0'11	10
1872'21	+ 4'0	- 0'60	3	84'19	- 0'4	- 0'25	10
	Hall (26 in.).			88'97	+ 0'5	- 0'10	5
1872'24	+ 1'2	+ 0'11	5	90'27	0'0	- 0'05	3
75'28	- 1'0	- 0'18	4		Pritchett (12 in.).		
76'22	- 0'5	+ 0'04	6	1878'19	+ 2'5	+ 0'45	5
77'26	- 0'2	- 0'05	5	79'15	+ 0'6	+ 0'24	5
78'24	+ 0'1	- 0'02	5		Hough (18 in.).		
79'20	+ 0'4	+ 0'03	6	1880'17	+ 2'1	- 0'37	3
80'25	+ 0'5	+ 0'09	8	81'26	+ 0'3	- 0'30	5
81'26	+ 0'3	+ 0'10	6	82'13	+ 0'4	- 0'27	9
82'23	+ 0'1	+ 0'12	7	83'12'	0'3	- 0'13	11
83'21	- 0'6	+ 0'13	6	84'18	- 0'2	- 0'15	11
84'23	+ 1'0	+ 0'18	8	85'20	- 0'7	- 0'14	10
85'27	+ 1'5	- 0'02	8	86'14	- 1'0	- 0'31	12
86'22	+ 1'3	- 0'06	6	87'19	- 1'2	- 0'03	7
87'23	- 0'5	- 0'24	4		Bigourdan (12 in.).		
88'24	0'0	- 0'20	3	1880'18	- 0'6	- 0'31	4
				81'14	- 0'8	+ 0'08	3
				82'13	- 0'2	+ 0'18	3

<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>	<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>
Frisby.				See (24 in.).			
1881'17	+1'7	+0'17	6	1897'80	-0'2	+0'61	4
82'18	-0'4	+0'38	6				
83'17	+1'5	+0'63	7	Hussey (36 in.).			
Perrotin (30 in.).				1899'09	+1'2	-0'11	5
1884'05	-1'2	+0'95	6	1900'41	+0'6	-0'15	3
				02'18	+0'4	-0'16	6
Young.				Barnard (40 in.).			
1881'18	+1'6	-0'13	6	1899'85	+1'5	+0'07	3
84'27	-0'3	+0'08	4	1902'01	+1'6	+0'07	2
85'11	+0'3	-0'06	8	03'09	-0'4	+0'04	4
86'11	-0'3	+0'01	3	03'81	-0'4	-0'04	2
87'14	+0'2	+0'26	4	04'85	+0'9	+0'14	3
Aitken (36 in.).				06'96	+0'3	+0'10	3
1896'83	-0'1	-0'09	4	08'17	-0'1	+0'28	6
97'16	+1'6	+0'03	4	09'14	-0'4	+0'13	10
97'81	+0'9	+0'01	4	10'11	-0'8	+0'10	8
98'16	+1'9	+0'10	2	11'00	-0'2	+0'17	7
98'73	+0'7	-0'08	3	12'07	+0'3	+0'28	5
99'22	+0'1	-0'02	2	13'05	+0'2	+0'06	15
99'90	-1'7	-0'17	5	14'15	-0'4	+0'04	5
1901'25	+0'5	-0'08	3	15'99	-0'4	+0'02	16
02'25	+1'0	-0'17	3	16'95	+0'3	-0'03	20
03'03	-0'1	-0'14	3	Olivier (26 in.).			
04'95	-0'5	-0'04	3	1906'11	+0'6	-0'45	3
06'08	-0'5	+0'03	2	16'86	+0'5	-0'20	10
07'01	-0'3	+0'06	2	Doolittle (18 in.).			
08'20	-0'2	+0'13	2	1908'22	+1'6	+0'16	4
09'25	-0'7	+0'04	2	Fox (18 in.).			
10'18	-0'8	+0'06	2	1910'10	+1'5	+0'49	6
11'21	-1'5	+0'04	3	11'00	+2'2	+0'37	7
12'08	-1'0	-0'03	2	11'92	+2'0	+0'28	5
Schaeberle (36 in.).				13'06	-1'2	+0'35	4
1896'85	+0'5	-0'05	4				
97'84	+1'7	-0'08	3				

<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>	<i>t.</i>	$\Delta\theta.$	$\Delta\rho.$	<i>n.</i>
Jonckheere (14-28 in.).				Innes (9 in.).			
1911'10	0°0	-0''56	4	1912'15	+0°4	+0''19	6
14'77	+0°8	-0°48	4	13'16	-1°3	-0°09	4
15'07	+1°1	-0°43	6	14'19	-0°8	+0°01	4
16'11	-0°7	-0°10	6	15'18	-0°8	+0°03	4
17'23	+1°0	-0°26	3	16'21	+0°5	+0°12	4
18'16	-0°1	-0°38	13	17'18	+0°4	+0°37	4

All the measures have been reduced for precession to the equinox of 1900, to which the elements of the orbit are referred.

It is difficult to separate the very large personal equation shown by these series from the actual defects in the orbit. The computed distances are too large for Burnham, Hough, Oliver, and Jonckheere. They are too small for Holden, Barnard, and Fox. Before periastron, Hall's fine series suit the orbit well, and the apparent systematic error given by Holden in angle and distance is not supported by Burnham, so that there is no apparent defect on that side. After 1894'10, that is, after periastron passage, we have two remarkable series of Aitken and Barnard. For these two observers the means which were obtained on two nights only have exceptionally been included. On this orbit, Barnard's measures appear to be systematically large by 0''10. It is interesting to note that the investigation on the measures of the satellite of Neptune by David Gibb* showed that the results from the Greenwich photographs were systematically 0''11 smaller than those obtained by Professor Barnard. There is, however, apart from this, a small increase in the distance in 1908, which is entirely supported by Aitken's measures. This and the curious run of the residuals in these last observations have led me to suspect the existence of a perturbing body. It is first necessary to ascertain how far these small deviations can be reduced by a new analytical investigation of the orbit, and this has been undertaken.

To save space the observed places were not given in the preceding table. The relative apparent position of the companion is given below for every year during a complete revolution, so that the place may be obtained for any time by subtracting fifty years from the time when necessary.

The computed positions are for the equinox 1900. For strict comparison the observations should be corrected by the following quantities:—

<i>t.</i>	Corr.	<i>t.</i>	Corr.
1860	+0°22	1910	-0°06
1870	'17	1920	'11
1880	'11	1930	'17
1890	+°06	1940	-°22

* *Proceedings of the Royal Society of Edinburgh*, 29, 517 (1909).

<i>t</i>	θ	<i>p</i>	<i>t</i>	θ	<i>p</i>
1894 ¹⁰	249 ⁰ 2	2 ⁷ 74	1919 ¹⁰	69 ⁰ 1	11 ¹ 15
95 ¹	222 ⁷	3 ² 23	20 ¹	67 ²	11 ² 26
96 ¹	202 ¹	3 ⁶ 3	21 ¹	65 ³	11 ³ 32
97 ¹	184 ⁸	3 ⁸ 4	22 ¹	63 ⁵	11 ³ 35
98 ¹	169 ⁶	4 ¹ 10	23 ¹	61 ⁷	11 ³ 35
99 ¹	156 ⁴	4 ⁴ 2	24 ¹	59 ⁸	11 ³ 33
1900 ¹	145 ¹	4 ⁷ 7	25 ¹	57 ⁹	11 ² 26
01 ¹	135 ⁵	5 ¹ 15	26 ¹	56 ⁰	11 ¹ 16
02 ¹	127 ³	5 ⁵ 8	27 ¹	54 ⁰	11 ⁰ 2
03 ¹	120 ²	6 ⁰ 4	28 ¹	52 ⁰	10 ⁸ 1
04 ¹	114 ¹	6 ⁵ 0	29 ¹	49 ⁹	10 ⁵ 6
05 ¹	108 ⁸	6 ⁹ 7	30 ¹	47 ⁷	10 ² 5
06 ¹	104 ²	7 ⁴ 0	31 ¹	45 ⁴	9 ⁹ 6
07 ¹	100 ¹	7 ⁸ 1	32 ¹	42 ⁹	9 ⁵ 9
08 ¹	96 ⁴	8 ² 2	33 ¹	40 ²	9 ¹ 6
09 ¹	93 ⁰	8 ⁶ 1	34 ¹	37 ²	8 ⁶ 9
10 ¹	89 ⁹	8 ⁹ 6	35 ¹	33 ⁹	8 ¹ 6
11 ¹	87 ⁰	9 ³ 2	36 ¹	30 ⁰	7 ⁵ 5
12 ¹	84 ³	9 ⁶ 1	37 ¹	25 ⁴	6 ⁸ 6
13 ¹	81 ⁸	9 ⁹ 1	38 ¹	19 ⁸	6 ⁰ 9
14 ¹	79 ⁵	10 ² 0	39 ¹	12 ³	5 ² 7
15 ¹	77 ³	10 ⁴ 5	40 ¹	2 ⁰	4 ³ 8
16 ¹	75 ²	10 ⁶ 7	41 ¹	346 ³	3 ⁴ 8
17 ¹	73 ¹	10 ⁸ 5	42 ¹	321 ¹	2 ⁷ 4
18 ¹	71 ¹	11 ⁰ 1	43 ¹	284 ⁶	2 ⁴ 6
19 ¹	69 ¹	11 ¹ 5	44 ¹	249 ²	2 ⁷ 4

Greenwich:

1918 June 9.

